VAN NOSTRAND'S

ENGINEERING MAGAZINE.

NO. CLXX.—FEBRUARY, 1883.—VOL. XXVIII.

THE CONSTRUCTION OF ELECTRO-MAGNETS.

By TH. DU MONCEL.

Translated from the French for VAN NOSTRAND'S ENGINEERING MAGAZINE.

II.

If in the problem now under discussion the force of electro-magnetic attraction is to be expressed as a weight, it is plain that the formula

$$\frac{E^{\frac{9}{2}}}{R^{\frac{9}{4}}} \cdot \frac{Q}{f^2}$$

will represent this force (which we will call P), only by applying a coefficient K, which expresses the ratio $\frac{F'}{P'}$ deduced from the standard electro-magnet, whose attraction at a millimeter distance is

$$\frac{0.002297}{26.85} = 0.00008555.$$

The quantity F represents in the standard magnet the value $I^2 t^2 c^{\frac{3}{2}}$, and if we assume

$$\frac{ne^{\frac{9}{2}}}{(n\rho+r)^{\frac{9}{4}}} \frac{Q}{f^2} = PK$$

we can easily deduce the value of ne, which will be

$$ne = \sqrt{n\rho+r} \sqrt[3]{\sqrt[3]{\frac{f^4 P^2 K^2}{Q^2}}}$$

As Q and K are constants they may be removed from under the radical sign, and the expression easily reduces to

$$\frac{n^2 e^2}{n\rho + r} = \left(0.0225 \sqrt[3]{\sqrt[3]{f^4 P^2}}\right)^2$$

The quantity in parenthesis which may readily be calculated, may be represented by A, and we obtain

$$n = \frac{A^2\rho}{2e^2} + \sqrt{\left(\frac{A^2\rho}{2e^2}\right)^2 + \frac{A^2 r}{e^2}}.$$

The values of ne and $n\rho$ being determined, it becomes easy to calculate the dimensions of an electro-magnet by means of the formulas already given.

In the preceding calculations the resistance of the circuit is supposed great enough to require that the battery employed should be arranged for *intensity;* but if the resistance is so small that the battery may be arranged for *quantity*, the calculations will require some modification. Two cases present themselves; the resistance R may be nothing; or it may be simply less than nr. In the first case we cannot obtain determinate values for a and b (the first representing the number of rows of cups in the battery, and b the number of cups in each

row), for the internal resistance of the battery which is then $\frac{a}{b}\rho$, and the resistance H of the helix, being two indeterminate quantities which should be equal, we can assign values at pleasure between the limits corresponding to the values arising from the condition of the battery being arranged entirely for intensity or entirely for quantity.* But we can obtain the value ab or the total number n of the elements by the preceding formulas, which give

$$ab=\frac{\rho}{e^3}\,0.000506\,\sqrt[3]{\sqrt[3]{P^4 f^6}}$$

or

$$ab=\frac{\rho}{e^3}\left(0.0225\,\sqrt[3]{\sqrt[3]{P^2 f^4}}\right)^2$$

Under these conditions the value of c is determinate, and the formula gives

$$c=\frac{e\sqrt{ab}}{\sqrt{\rho}}=0.173. \tag{13}$$

In the second case, as the resistance ought to be equal to $\frac{a}{b}\rho+r$ in order to satisfy the conditions of maximum effect in the magnet, and as in order to give the resistance of the circuit a minimum value, corresponding to a maximum for I, a resistance based upon that of the electro-magnet, it is necessary that $\frac{a}{b}\rho=r$, we shall be able to determine a and b whenever we know ab or n. This will be determined by the preceding equation, which gives the value of ab, since the value $\frac{a}{b}\rho+r$ becomes then equal to $2\,\frac{a}{b}\rho$. We have then

$$a=\sqrt{\frac{nr}{\rho}},\ \text{and}\ b=\sqrt{\frac{n\rho}{r}},\ \text{or}\ \frac{n}{a}.$$

If, instead of a simple circuit we have a circuit with x branches starting from the poles of the battery, and upon each of these branches there be placed an electro-magnet, all having the same resistances and the same dimensions, the intensity of the current upon each circuit would be

$$\frac{aE}{x\frac{a}{b}\rho+H},\ \text{or}\ \frac{bE}{2x\rho},$$

and the value of ab or n would be the same as in the case of the simple circuit multiplied by x.

In case the dimensions of the electro-magnet are all given, and it is desired to determine the size and arrangement of the battery to obtain the force P upon each of the x derived circuits:—

From the assumed conditions, the value of H in the equation

$$I=\frac{aE}{x\frac{a}{b}\rho+H}$$

is known; and we also know that for maximum effect $x\frac{a}{b}\rho$ should be equal to H. Furthermore, we know that we should have

$$I^2 t^2 c^{\frac{3}{2}}=PK.$$

Now, as the quantities t^2 and $c^{\frac{3}{2}}$ can be easily calculated, since c is given, the value of I is readily found.

$$I=\sqrt{\frac{PK}{t^2 c^{\frac{3}{2}}}};$$

from which we deduce

$$a=\frac{2IH}{E}\ \text{and}\ b=\frac{ax\rho}{H}.$$

V.—Numerical Examples, Applying the Foregoing Formulas.

To apply the preceding formulas to some practical cases, we will at first suppose that we wish to determine the best

* This appears clear from the following calculation: The attractive force of an electro-magnet may be represented by I^2H, and when the battery is arranged in series, this expression becomes

$$A=\frac{a^2E^2H}{\left(\frac{a}{b}\rho+H\right)^2}=\frac{a^2E^2H}{\frac{a^2}{b^2}\rho^2}$$

Now, suppose the battery to consist of 24 elements, each having a resistance of 12, then the electromotive force being equal to 1, when the elements are united for intensity,

$$A=\frac{1\times 288}{4\times 12^2}=0.5,$$

when united for quantity,

$$A=\frac{24^2\times 1\times 0.5}{4\times 12^2}=0.5,$$

and when arranged so that $a=b$, then

$$A=\frac{24\times 1\times 12}{4\times 12^2}=0.5,$$

all the values being identical.
See also Chapter IX of this essay.

dimensions for an electro-magnet to be worked by a single Bunsen cup of medium size, upon a circuit having no other resistance than that of the helix. We will call the electromotive force 1.86, a Daniel element being the unit and the resistance that of 57 meters of telegraph wire, which is 0.57 ohm. These first given quantities exhibit the necessity of knowing the constants for the different batteries in use. They will be given further on.

From formula 10 we shall have for c the diameter of the iron core:

$$c=\frac{1.86}{\sqrt{57}}0.172175=0.^{m}0424.$$

Each of the branches will have then a diameter of $0.^{m}0424$ and a length of $0.^{m}255$. The diameter g of the wire of the helix will be, by formula (7),

$$g=\sqrt{1.4\sqrt{\frac{0.0424^{2}}{57}0.00020106}}=0.^{m}004865$$

which includes the insulating cover. The wire alone would be $0.^{m}00336$ which is the value of g divided by f or 1.4.

The length of the wire would be by formula (5) 243.^{m}, and the attractive force expressed as a weight at one millimeter, from the formula

$$A=\frac{I^{2}l^{2}c^{\frac{3}{2}}}{0.0000855}$$ will be 23.^{k}112.

If the quantity R is expressed in ohms instead of in meters of telegraphic wire, the constant of formula (7) should be 0.0000020106 instead of 0.00020106, so that the value of g would be

$$g=\sqrt{f\sqrt{\frac{c^{3}}{R}0.0000020106}}.$$

The number of spiral turns is to be calculated by formula (6).

Suppose now that this electro-magnet be inserted upon a line of 100 kilometers, which with the resistance of the necessary battery composed of 20 Daniell elements will form a circuit of 118620 meters resistance (measured in telegraph wire); we shall have

$$c=\frac{20}{\sqrt{118620}}0.172175=0.^{m}01,$$

or in electrical units

$$c=\frac{21.58\text{ volts.}}{\sqrt{1186\text{ ohms.}}}0.015057=0.^{m}01.$$

Each of the branches will then have a diameter of $0.^{m}01$, and a length of $0.^{m}06$. The diameter of the wire will be $0.^{m}0002597$, including the covering, and $0.^{m}0001583$ without it, and the length will be 1116 meters. The attractive force equals 26.85 grams.

The different formulas show why electro-magnets that are introduced into long circuits should have small dimensions and have helices of fine wire; and why, on the contrary, for short circuits, the dimensions should be larger, and the battery be arranged for quantity.

The dimensions now to be proposed for an electro-magnet worked by a single Bunsen cell will appear to most people rather large; but they would be still greater if iron of a lower degree of magnetic saturation were to be employed. Thus a tubular electro-magnet having a diameter of $0.^{m}10$, a length on each arm of $0.^{m}30$, thickness of tube $0.^{m}01$, has been made to support a weight of 160 kilograms when charged by a single Bunsen cell. The helix being a copper wire 4 millimeters in diameter (not including the covering), 182 meters long and wound in 482 spiral turns. The poles of the magnet were not even furnished with an iron stopper.

With an ordinary electro-magnet $0.^{m}02$ in diameter, wound with copper wire one millimeter in diameter, the writer has never been able to obtain with a single Bunsen cell more than 12 kilograms of attractive force at contact with the poles.

We will assume now that we are to obtain a given force on a circuit of given resistance; that the attractive force is to be 273 grams at a millimeter distance; the circuit to have a resistance of 500 ohms, 50 kilometers of telegraph wire), to be charged with a single bichromate cell. In the proposed battery (potassium bichromate with sand) the electromotive force is 2 and the internal resistance about 10 ohms.

Applying the formulas in Chapter IV, we have

$$A=0.0225\sqrt[3]{\sqrt[3]{1.37^{4}\times 273^{2}}}=0.09$$

and $A^{s}=0.0081$.

$$n=\frac{8.1}{8}=\sqrt{\left(\frac{8.1}{8}\right)^2+\frac{0.0081\times 50000}{4}}$$
$$=11.125$$

from whence

$$c=\frac{11.125\times 2}{\sqrt{61125}}\times 0.173=0.^{m}01553.$$

This gives for the length of each helix 0.^{m}0932. Diameter of wire without covering, 0.^{m}0002842. With its covering, 0.^{m}0003894. Length of the wire, 1861 meters. Number of turns, 19078. Intensity of current, 0.0001859. The value $c^{\frac{3}{2}}=0.001935$.

Squaring the values of I and t and multiplying by $c^{\frac{3}{2}}$ we get for the electro-magnetic force 0.0243778517. Comparing this value with that of the standard electro-magnet, which is 0.002297, we find the ratio to be 10.6, which is very near the ratio demanded. The attractive force in grams of the standard being 26.85, and the force required above being 273 grams.

We will now suppose it is desired to obtain a force of 200 grams on each of six electro-magnets all connected with a battery of the same kind as the preceding.

The total number of cells will be given by the formula:

$$ab=\frac{6\times 1000}{4}\times 0.000506\sqrt[3]{\sqrt{200^4\times 1.4^2}}$$
$$=10{,}778,$$

or say 11 cells. But as this number is not readily divided into groups, we will take 12. With this number of cups we can arrange them in 3 rows of 4 cups, or in 2 rows of 6 cups each, but in either case we shall have

$$c=\frac{\sqrt{12}}{\sqrt{1000\times 6}}\times 2\times 0.173=0.^{m}0153$$

and consequently the length of each branch of the electro-magnets will be 0.^{m}0918, or about a tenth of a meter.

Now we will compare the values obtained under the supposition of the two different arrangements of the 12 cells.

1st, when $a=3$ and $b=4$.

$g^2=0.000000560075.$
$g=0.0007484.$
$\frac{g}{f}=0.0005346.$
$H=482^{m}$
$t=5015,\ t^2=25150225.$
$I=0.000666.$
$I^2=0.000000443556.$
$c^{\frac{3}{2}}=0.001892.$
$I^2t^2=11.155533.$
$I^2t^2c^{\frac{3}{2}}=0.021084.$
$A=247$ grams.

2d, when $a=2$ and $b=6$.

$g^2=0.00000083986.$
$g=0.000916.$
$\frac{g}{f}=0.0006543.$
$H=321^{m}.$
$t=3345,\ t^2=11189025.$
$I=0.001,\ I^2=0.000001.$
$c^{\frac{3}{2}}=0.001892.$
$I^2t^2=11.189025.$
$I^2t^2c^{\frac{3}{2}}=0.0211686353.$
$A=247$ grams.

It is not necessary, however, to conclude that in order to maintain a good condition of saturation in the magnets, that we ought to vary their dimensions according to the resistance of the exterior circuit. We can for some applications keep to the same dimensions for the magnet, by varying the force of the battery and the dimensions of the wire of the helix. Since the electromotive force of the battery employed, for the same diameter of electro-magnet, and a given electro-magnetic force is proportional to the square root of the exterior resistance of the circuit, we may conclude that if we augment the electromotive force E in a proper ratio, we may as the resistance of the circuit increases, continue the use of an electro-magnet of the same diameter and having the same force; consequently, if instead of inserting our standard magnet of 0.^{m}01 diameter in a circuit of 100 kilometers, we introduce in a circuit of 400 kilometers with double the number of cells, we shall have very nearly the same effect produced. But it would be necessary to change the dimensions of the wire, for the diameter of the magnet remaining the same, the diameter of the wire should vary in the inverse ratio of the fourth root of the resistance of the exterior circuit. It would be necessary, then, in

the example cited that this diameter should be to the diameter of the wire of the standard magnet 0.m0001583.

as $\sqrt[4]{100000} : \sqrt[4]{400000}$,

or as 17.8 : 25.1,

which gives 0.000112. We see that for telegraph relays the assigned dimensions may be preserved.

It has been asserted above that in doubling the number of elements of a battery, we shall obtain *nearly* the same force upon a circuit of 400 kilometers as previously upon a circuit of 100 kilometers. The qualification *nearly* is necessary, for as the resistance of the circuit, the resistance of the battery is also counted, and as the latter varies with the number of elements employed, the real resistance of the circuit is 437240 meters instead of 400000 (400km). Now this is a little less than 4 times 118620m which represents the resistance of the first circuit, so that to obtain the same effect in the two cases it would be necessary that the number of elements (20) should be raised to 38.4 (say 39) instead of 40.

We will now consider the case of two small electro-magnets, each having 2200 meters resistance, of No. 15 wire, and connected with a Daniell battery of 8 cups arranged for intensity. Experiment proves that the force obtained from each would be 70 grams at a millimeter distance. Consequently the force developed by the pair would be 140 grams. Now if in place of the two magnets only one had been employed, the force would have been 200 grams. There is then a loss of 60 grams by the use of the second magnet, although both of them are connected directly with the poles of the battery. It would seem at first sight as though there were some defect in the electrical connections, but we will proceed to show that calculation verifies the above result.

Applying Ohm's formulas to the two cases, we find: 1st, that in the case of the single electro-magnet, if we represent the electro motive force of the Daniell cell by 5973, the intensity of the current will be 4.95; 2d, that, in the case of the two electro-magnets forming two branches the intensity is 2.79.

The forces of the electro-magnets under these two arrangements are as the squares of these numbers 4.95 and 2.79, and if we estimate the force of the electro-magnet in a simple circuit at 200 grams, the force of the two others would be given by the formula

$$\frac{\overline{2.79}^2 \times 200}{\overline{4.95}^2} = 64 \text{ grams.}$$

Consequently the force of the pair of magnets would be 128 grams against 200 grams for the single magnet.

The result arises from the arrangement of the battery which was not adjusted to accord with the exterior resistance, since its resistance is nearly four times as great as the latter, and is again less than the resistance of the circuit containing the two magnets. The formula

$$\frac{n\mathrm{E}}{2n\rho + \mathrm{H}},$$

which represents the intensity of the current upon each branch, and which, in case of a battery arranged in series, gives

for condition of maximum effect $2\frac{a}{b}\rho = \mathrm{H}$,

or $\frac{a}{b}\rho = \frac{\mathrm{H}}{2}$, shows that in this case the resistance of the battery ought to be half that of each circuit, consequently the number a of the elements in tension should be determined by the formula:

$$a = \sqrt{\frac{8 \times 2200}{2 \times 931}} = 3.074.$$

As we cannot divide a cell of a battery, and as the plan requires that the numbers expressing values of a should be divisors of n, the battery which best lends itself to the several conditions of maximum is the one which most readily admits of division into various groups, and we see that in the case before us the conditions require 3 cups in tension and 3 in quantity. Now under these conditions, the attractive force of the single electro-magnet would be 267 grams, and of the two magnets 316 grams. There is then a gain in the use of the two magnets.

The foregoing examples show the importance of making calculation precede the construction of electro-magnets and the arrangement of the battery.

VI.—Experimental Verification of the Laws of Electro-Magnetism.

The formulas given in the preceding chapters have all been verified by the writer's experiments, except such as are generally adopted and which present difficulties in verification as we shall see forthwith.

This experimental work seemed the more important, as formerly many sicentists would not accept the formulas without discussion, and furthermore mathematical calculations are not always convincing. The writer has made many experiments to prove the truth of his deductions and with results as follows:

1st. The experimental demonstration of the principle which establishes the rule that the resistance of the electro-magnet should be equal to that of the exterior circuit is quite difficult by reason of the difficulty of finding wire of different sizes whose conductivities are exactly proportioned to the squares of the diameters. The experiments tried gave contradictory results, and it is proper to say, some that were contrary to the theoretical conclusions, which led in the beginning of these researches to some hesitation in accepting them, although they had been already generally accepted. Meanwhile as the other conclusions have been confirmed by experiment, and as the ill success of those experiments relating to this subject may have been due to accidental causes, the principle above stated has been admitted in those cases in which it is really applicable: that is to say when the size and length of the spool or helix having been fixed, it is required to know the size of the wire which should be employed to correspond to a circuit of given resistance. It is a case in which the elements of the construction have been already determined, since the size of the iron core should in the first place be proportioned to the intensity of the current, and since the thickness of the helix should be determined by another important principle.

This is not the same problem as the determination of the resistance of the circuit upon which a given electro-magnet or galvanometer can be profitably employed, without having previously considered its dimensions. In this latter case, the conditions of maximum effect as established by the writer show that, *the resistance of the electro-magnet should be to the resistance of the exterior circuit, as the thickness of the helix plus the diameter of the core, is to the thickness of the helix.*

This rule has been established by the following experiments:

Upon a spool whose length between the ends was 0.m061, and with a tube 0.m011 in diameter there was wound with great care two lengths of 60 meters each of No. 15 wire, also another length of 57¼ meters which formed a third helix. These three helices presented their ends outward each distinct from the other so that they could be used separately or together. The first offers a resistance of 1800m of telegraph wire; the second had nearly the same so that the two joined made a resistance of 2160 meters; and finally, the third helix added to the two first furnished a total resistance of 3200 meters.

In subjecting this electro-magnet to the test of the writer's electro-magnetic balance, and applying to that pole only which was enclosed by the helix, and exciting the magnet by the current from a Leclanché battery of 3 cups, each with a resistance of about 400m, the following results were obtained:

Resistance of exterior circuit.		Helix A. 1080m.	Helix B. 2160m.	Helix C. 3200m.
m.	m.	gr.	gr.	gr.
1200+	0 (1200)	F=112	F′=122	F″=112
1200+	400 (1600)	F= 73	F′= 92	F″= 95
1200+	1000 (2200)	F= 47	F′= 66	F″= 70
1200+	2000 (3200)	F= 27	F′= 43	F″= 50
1200+	3000 (4200)	F= 17	F′= 29	F″= 36
1200+	4000 (5200)	F= 12	F′= 22	F″= 28

The attractive forces F, F′, F″ were measured at the distance of one millimeter from the pole.

Now this table shows that the helix B, whose conditions of resistance most nearly accord with the theoretical conditions,

$$\left\{\frac{2160}{1+\frac{c}{a}}=982\right\}$$

affords the maximum effect, and it is only when the resistance of the exterior circuit becomes 1600m or $\frac{3200}{2}$ that the

helix of highest resistance C exhibits a preponderance.

With 2 cups of the battery and an exterior resistance of 800^{m} (represented by the battery), the helix B gave a force of 60 grams, while A and C each gave 57 grams. With a single cup and its resistance of 400, the helix A had the advantage, and the forces were 21 grams for A, 19 for B and 17 grams for C.

These experiments repeated with the galvanometer were yet more decisive as the reader may readily verify by consulting the account published by the author in *Comptes rendus des Séances l'Académie des Sciences*, Aug. 13th, 1877.

2d. In order to demonstrate that upon a branched circuit, the resistance which is to serve as the basis of calculation of that of an electro-magnet is the total exterior resistance *taken inversely*, that is to say as though the generator had been substituted for the electro-magnet —a sensitive Ruhmkorff galvanometer was inserted in a circuit having a very feeble generator. The galvanometer had two multipliers, one of 733 kilometers resistance, and the other of 237^{km}. A rheostat was added to this circuit, and a second one to a branch joining the two extremities of the galvanometer wire.

Calling R the circuit of the electrometer, u the galvanometric derivation, l the derived circuit in which the galvanometer was inserted, the equation expressing the condition of maximum effort was

$$l+\frac{Ru}{R+u}=\frac{\pi ba^2}{g^2}.$$

It is true that l cannot well influence this calculation, since the points of branching were the two extremities of the galvanometer; but by means of the two rheostats, the resistances u and R could be so varied as to present a combined *total* resistance, which was equal to, or greater, or less than the less resistant of the multipliers of the galvanometer; so that it sufficed, R being given, to calculate u by the formula

$$u=\frac{RH}{R+H},$$

H representing the resistance of the multiplier.

The results obtained after taking all precautions to preserve proper conditions were as follows:

Derived circuits. km. km.	Total resistance. km.	Multiplier 237^{km}. °	Multiplier 733^{km}. °
R=512+272 / u=86	78	I=27½	I′=24½
R=512+272 / u=128	110	I=32½	I′=30
R=512+272 / u=200	160	I=36¼	I′=36¼
R=512+272 / u=256	193	I=40	I=41
R=512+272 / u=512	309	I=46	I=51
R=512+272 / u=200	145	I=36	I=33¼
R=512+272 / u=512	260	I=47	I=52

It will be seen by this table that, as was said above, the multiplier (resistance coil) of greatest resistance has the advantage even where the total resistance of the circuit outside of the galvanometer is nearly equal to the smaller resistance coil; and it is only when this total resistance falls below 160^{km} or 145^{km}, a point at which the two multipliers have the same sensitiveness that the multiplier of least resistance exhibits an advantage. As with the galvanometer experiment, the ratios of the two resistances (the exterior circuit and the resistance coil), ought to be according to the conditions of maximum deduced from calculation 1.9 and 2.425—the resistances of the exterior circuit where the multipliers could be applied to the best advantage, should be 98^{km} for the multiplier of 237^{km}, and 386^{km} for the multiplier of 733^{km}; and the point at which they become equally sensitive ought to correspond to a resistance which is a mean proportional between these two numbers. Now this mean proportional being 193, and as the number furnished by experiment is 160, we may conclude that the principle previously stated has been practically verified. It is capable of further proof by calculating the resistance of u, which would equal deflections for the two resistance coils. This resistance is given by the formula

$$u=\frac{R(t'H-tH')}{t(R+H')-t'(R+H)},$$

which gives under conditions established for the galvanometer experiment, 224^{km}, while the experiment indicates 200^{km}.

3d. The principle governing the thick-

ness of the magnetizing helix—that it should be equal to the diameter of the iron core—is readily verified. To demonstrate this experimentally, the writer took three magnets of the same length but of different diameters, as follows: 1st, 0.m02; 2d, 0.m01, and 3d, 0.m0065. Only one pole was brought to act upon the electro-magnetic balance; each spool was wound with No. 16 wire; 23 layers of 111 turns each, or a total of 2553 spiral turns. The wire was closely wound in each case, and the helices had a uniform thickness of 0.m01.

It follows, therefore, that the dimensions of the 2d magnet alone corresponded with the conditions of maximum effect as previously stated. This magnet had a resistance of 3200 meters, while the larger offered a resistance of 5200, and the smaller one 2800 meters.

The results of the experiments are exhibited in the following table. The battery consisted of from 1 to 3 Leclanché cells, each having a resistance of 400m. The magnetic attraction was estimated at a millimeter distance.

	Resistances of circuit.			Attractive force of the magnets. No. 1. 0.m02	No. 2. 0.m01	No. 3. 0.m0065
	m.	m.	m.	gr.	gr.	gr.
3 cells.	1200+	0	(1200)	76	112	86
	1200+	1600	(2800)	48	57	(44)
	1200+	2000	(3200)	43	(50)	39
	1200+	4000	(5200)	(28)	29	22
	1200+	10000	(11200)	13	10	8
2 cells.	800+	0	(800)	38	57	46
	800+	2000	(2800)	22	26	(20)
	800+	2400	(3200)	20	(22)	16
	800+	4400	(5200)	(14)	13	10
	800+	10000	(10800)	7	5	4
1 cell.	400+	0	(400)	12	18	15
	400+	2400	(2800)	7	7	(6)
	400+	2800	(3200)	6	(6)	5
	400+	4800	(5200)	(5)	4	4
	400+	10000	(10400)	3	2	2

The table clearly shows that for the same resistance of exterior circuit, and with sufficient intensity of current, the advantage is with the magnet whose diameter equals the thickness of the helix. This advantage is maintained so long as the forces produced through the circuits of different resistances are properly proportioned to the resistances of the electro-magnets; and such conditions only are considered in the formula. It is only when the electric force is so feeble that the increase of magnetic action with the increase of diameter is but slightly marked, that the superiority of magnet No. 2 is affected. This is in accoad with Müller's law, for the attractive forces of the magnets are proportional to their diameters only when the magnets are charged nearly to the point of saturation. By the term saturation is meant that magnetic state which the magnet would retain if it were hardened steel instead of iron. When the force developed is much below this, it is dependent chiefly upon the intensity of the current, and that is naturally the strongest on the circuit of least resistance.

The numbers in parenthesis in the preceding table indicate the forces corresponding to maximum conditions relative to the circuit, and these conditions have been established on the hypothesis that the resistance of the exterior circuit was equal to the resistance of the helix, and in the construction of these electro-magnets a given thickness of helix was the starting point. If it had been based upon maximum conditions of a given electro-magnet without consideration of dimensions the preceding law would assert itself still more, for the maximum resistances of the exterior circuit become 2600m for the larger magnet, 1600m for No. 2, and 1400m for No. 3. The attractive forces of the three magnets then become:

	With 3 cells.	With 2 cells.	With 1 cell.
For magnet No. 1..	50 gr.	25 gr.	8 gr.
" " 2..	94	46	14
" " 3..	79	41	13

VII.—Effects upon Electro-magnets of Saturation more or less Complete.

As was said in a previous chapter, the laws governing electro-magnets can not all be stated with desirable precision by reason of the variable resultant effect of saturation of the soft iron core; but this cause of perturbation is far from being as prominent as some scientists would have us believe, and it interferes with the definite results which they have deduced, much less than the effects of polarization have upon calculated results of electric currents. During the last researches upon electro-magnets, the writer wished to assure himself of the importance of this cause of perturbation,

and a large number of experiments were undertaken which seem to be of sufficient interest to be related here, inasmuch as the rapid extension of the application of electro-magnetism renders it daily more important that the best working conditions of electro-magnets should become known.

In the second chapter of this essay some general principles were stated, deduced from the author's experiments, but not much stress was laid upon these experiments as they were not conducted with reference to any law outside of those formulated by Dub and Müller; but it is important that a more definite explanation should now be given.

First it will be shown that having determined that the number 11 is the proper multiplier by which to determine the length of the magnet after its diameter is fixed, it will follow that the attractive force will increase in proportion to the length. So that starting with a given length of wire to represent the resistance of the exterior circuit, and winding it upon electro-magnets of different diameters so as to furnish each with a thickness of helix equal to the diameter of the core, it would be necessary that the lengths should be different and that should satisfy the following conditions:

$$\frac{2\pi bc^2}{g^2}=\frac{2\pi b'c'^2}{g^2}, \quad \text{or } bc^2=b'c'^2,$$

so that the lengths would be inversely proportioned to the squares of the diameters. In this case the factor m (ratio of length in diameter) is no longer constant, and becomes proportional to the cubes of the diameters.

But then the law which directs that the electro-magnetic force should be proportional to the square of the number of turns of the helix wire, multiplied by the $\frac{5}{2}$ power of the diameter is not applicable, and it becomes necessary to resume the consideration of it so as to make a comparison of such forces among themselves, according to the law which states that these forces are proportional to the squares of the number of turns of the wire, multiplied by the diameters of the cores and the square roots of their length. We shall then have for the same intensity of current:

$$\frac{A}{A'}=\frac{c^2b^{\frac{5}{2}}c\sqrt{b}}{c'^2b'^{\frac{5}{2}}c'\sqrt{b'}}=\frac{c^3b^{\frac{5}{2}}}{c'^3b'^{\frac{5}{2}}}=$$

$$=\frac{c^3c'^5}{c'^3c^5}=\frac{c'^2}{c^2}=\frac{b}{b'},$$

which shows that *the forces are in the inverse ratio of the squares of the diameters, or proportional to the lengths;* always supposing that the iron cores are in the proper condition of saturation to render the laws of Dub and Müller applicable.

In order to make known the modifications required to the laws formulated connecting the state of saturation with the length and diameter, three electro-magnets having diameters of 0.m008, 0.m007 and 0.m006 respectively, were wound with the same length of No. 12 wire, having a diameter of 0.m00059 including its cover. This length was 71.m47, and its resistance was equal to 722 meters of telegraph wire or 7.22 ohms; and the spools were so constructed that the thickness of the helix was in each case equal to the diameter of the core. The lengths of the spools were, consequently: the first, 0.m059; the second, 0.m077; and the third, 0.m098; and the number of turns of wire were respectively, 1470, 1677 and 1842. The results given in the table on next page were obtained with a Leclanché battery of the later pattern, varying from 1 to 4 cells, each having a resistance of 113m or 1.13 ohms. The attractive force was measured at the distance of a millimeter.

These results show that conformably to the theory, it is the longest electro-magnet which is generally the strongest, but is in *the same ratio as the length, only for electro-magnets of certain limits as to diameter, and for a certain intensity of current which corresponds probably to the saturation of the magnets.*

This intensity for electro magnets of 0.m098 and 0.m077 varies from 0.001082 0.001030, 0.00143 and 0.00113 in the four series of experiments; but in comparing the electro-magnets of 0.m098 and 0.m059, the proportion in question is no longer related to the intensity of the currents and the ratio of the forces is less than the ratio of the lengths. Further than this it may be remarked, that the

No. of cells.	Exterior resistances.	Force of magnet. 0m,098.	Ratios.	Force of magnet. 0m,077.	Ratios.	Force of magnet. 0m,059.
	m m m	gr.		gr.		gr.
4 Cells.	452+ 0 (452)	225	1,018	221	0,9210	240
	452+ 300 (752)	151	1,049	144	0,9660	149
	452+ 400 (852)	134	1,0635	126	0,9770	129
	452+1000(1452)	73	1,0900	67	1,0307	65
	452+2000(2452)	36	1,161	31	1,0333	30
	452+3000(3452)	21	1,235	17	1,0630	16
	452+4000(4452)	14	1,272	11	1,1000	10
3 Cells.	339+ 0 (339)	169	1,0432	162	0,953	170
	339+ 300 (639)	109	1,0792	101	1,010	100
	339+ 400 (739)	95	1,092	87	1,0117	86
	339+1000(1339)	50	1,163	43	1,0238	42
	339+2000(2339)	23	1,2105	19	1,0555	18
	339+3000(3339)	13	1,300	10	1.000	10
	339+4000(4339)	8	1,333	6	1,000	6
2 Cells.	226+ 0 (226)	100	1,111	90	1,000	90
	226+ 300 (526)	60	1,154	52	1,0196	51
	226+ 400 (626)	52	1,1818	44	1,0232	43
	226+1000(1226)	26	1,3000	20	1,000	20
	226+2000(2226)	11	1,375	8	1,000	8
	226+3000(3226)	6	1,500	4	1,000	4
	226+4000(4226)	4	2,000	2	1,000	2
1 Cell.	113+ 0 (113)	35	1,2070	29	1,036	28
	113+ 300 (413)	19	1,2666	15	1,071	14
	113+ 400 (513)	16	1,3333	12	1,0909	11
	113+1000(1113)	7	1,1666	6	1,200	5
	113+2000(2113)	2	1,0000	2	2,000	1
	113+3000(3113)	1	1,0000	1	"	0
	113+4000(4113)	0	"	0	"	0

forces of these electro-magnets is so governed by the state of saturation of the core that for the highest electric intensities, and upon circuits of lowest resistance, it is the shortest and largest electro-magnet that gives best results. Conversely the narrowest and longest magnet exhibits its advantage more and more in proportion as the electricity diminishes, whether this diminution is caused by lessening the number of cells of the battery, or by increasing the resistance of the circuit. The importance of such variations can be estimated by the relations of the forces exhibited in the second and fourth columns of the preceding table. It is readily understood why it should be so, as for very high intensities, the diameter of the longer electro-magnet is not in proper proportion to this intensity, and it has passed its point of saturation when the short and large magnet would have just attained it. On the other hand, the advantage of the magnet of small diameter, under the action of feeble currents, is explained by this consideration; that the magnetic mass of the electro-magnet being sufficiently great compared with the electric force, this force affects beneficially the greater number of the turns of the helix wire. We may conclude, then, that the dimensions to be given to an electro-magnet ought to depend essentially upon the electric force which excites it, and upon the resistance of the circuit in which it is to be placed. *When the circuit is long and the electric source is of feeble energy, the magnet should be long and of small diameter.* When, on the *contrary, the circuit is short and the electric current intense, the iron core should be of large diameter.*

This is expressed otherwise by the formula $c=\frac{E}{\sqrt{R}}0.173$, which is offered by the writer to aid in determining the diameters of electro-magnets when the other conditions are known.

In regard to the influence of the condition of the magnetic saturation of the iron, it is difficult to formulate its effects. Joule, Müller and Robinson all long ago recognized the fact, that at the commencement of the current and while

the magnetic state is yet far short of saturation, *the attractive force instead of increasing as the square of the intensity the current augments in a much higher ratio, exceeding the third or even the fourth power of the intensity.* But the fact is also verified, *that in proportion as the electro-magnetic force is developed this ratio diminishes rapidly to the point of saturation, remains stationary for a moment, and beyond this point diminishes till the forces are in simple ratio with the intensity of the current.*

If the magnetic force thus varies as it is being developed, similar results should be exhibited when it can be completely developed in magnets whose dimensions admit of different degrees of saturation for the same intensity of current. This hypothesis can be verified by the results of the experiments already cited.

Considering the case of the three magnets in the last example, in which the forces were developed by currents of different intensities; *we find that there is for each of them a degree of intensity for which the attractive force varies as the square of this intensity, and that above and below this limit it increases in a more or less rapid ratio. Also, that this limit varies with the dimensions of the electro-magnet.*

In preparing from these results the table which follows, we see: 1st, That it is the electro-magnet B which best conforms to the law of the squares of the electric intensities. 2d, That the larger magnet C affords the highest ratio of increase. 3d, That the smallest magnet A furnishes the lowest ratio. 4th, That the ratio of the forces for the three magnets varies rather more rapidly than it should to accord with the law of squares of the intensities, as the intensities become weaker, whether from reducing the number of cells or from increasing the resistance.

It is certain that these ratios, notwithstanding their larger value, correspond more nearly to the squares of the intensities than to their cubes. We have, for example, between the first and last experiment of the first series, 85600 instead of 19423; but we will remark that the three magnets have diameters not widely different; that they are nearly in the same condition as to saturation, and that the forces have been measured at the moment of maximum magnetization.

It is evident that we should not have obtained the same results with magnets differing more in their diameters, nor if the forces had been measured instantly at the moment of magnetization, and during a closing of the circuit. In such a case we should, perhaps, be able to get a higher ratio than 85600. Upon this principle is based the action of the sluggish electro-magnets employed in some of the printing telegraphs, and which are slow in action only because their mass being relatively large, considerable time is consumed in charging them.

The following table exhibits the ratios previously alluded to:

No. of cells.	Ratios of square of intensity of current.	Ratio of forces. A $0^m,098$.	Ratio of forces. B $0^m,077$.	Ratio of forces. C $0^m,059$.
4 Cells.	1,576	1,49	1,54	1,61
	1,797	1,68	1,75	1,86
	3 429	3,082	3,30	3,70
	7,309	6,25	7,13	8,00
	12,640	10,71	13,00	15,00
	19,423	16,07	20,09	24,00
3 Cells.	1,645	1,55	1,60	1,70
	1,896	1,78	1,86	1,97
	3,773	3,38	3,77	4,05
	8,323	7,35	8,52	9,44
	14,650	13,00	16,20	17,00
	22,753	21,12	27,00	28,33
2 Cells.	1,733	1,66	1,73	1,76
	2,022	1,92	2.04	2,09
	4,222	3,84	4.50	4,50
	9,670	9,09	11,25	11,25
	17,343	16,66	22,50	22,50
	27,242	25,00	45,00	45,00
1 Cell.	1,847	1,84	1,93	2,00
	2,187	2,19	2.42	2.55
	4,829	5,00	4,83	5,60
	11,527	17,50	14,50	28,00
	21,094	35,00	29,00	"
	33,529	"	"	"

From the numerical results given in this table, we are led to conclude that the law making the attractive forces proportional to the square of the intensities of the currents, holds good within certain limits, and under certain indications; and that the electro-magnets over which the currents are broken at short intervals, fail more or less. Now as this case is involved in the important question of

the magnetizing helix, it would seem opportune to consider it from this new point of view; not to establish the conditions of maximum effect which have been already determined, but to find out in what way they are to be modified to suit certain cases.

The question is this: When the electro-magnetic force increases in a more rapid ratio than that of the squares of the intensities of the current (say as the cubes for instance), should the resistance of the magnetizing helix be greater or less than that of the exterior circuit?

To answer this question it will suffice to change in the formula giving the value of electro-magnetic force the exponents of the values of the intensity, or in other words, change I^2 to I^3. We obtain then this expression:

$$8f^3\pi ba(a+c)=4q\mathrm{R}g^4$$

which, if g varies, gives for a maximum

$$\frac{\pi ba(a+c)}{g^2}=\frac{q\mathrm{R}g^2}{2f^3},$$

or

$$\mathrm{H}=\frac{\mathrm{R}}{2}.$$

If a is the variable quantity, the conditions of maximum correspond to the equation

$$\frac{q\mathrm{R}g^2}{f^3}=\frac{\pi b\,a\left(2a+\frac{c}{2}\right)}{g^2}.$$

These two equations show that when the forces are proportional to the cubes of the intensity of the current, *the helices should offer less resistance than the exterior circuit; only half as much if g is the varible quantity, and in the ratio of $a+c$ to $2a+\frac{c}{2}$ when a is the variable.*

We may conclude, then, that upon circuits subject to frequent interruptions, the resistances offered by the magnets should be lessened in proportion as the intervals of closing the circuit are shortened. And it is for this reason as much as for that of defective insulation and of extra currents, that Hughes first, and subsequently telegraph engineers, reduced considerably the resistances of magnets upon long circuits.

It would be equally interesting to know in what way the resistance of the electro-magnet should be modified in the case where the point of saturation being passed, the force increases less rapidly than the square of the intensity of the current. If we suppose that this increase is simply in the ratio of the intensity, then there is no maximum possible, and we can with advantage increase the resistance of the magnets beyond the limits which have been assigned.

In seeking to determine how the conditions of maximum effect which involve the thickness of the helix should be modified to suit the conditions of defective saturation of the magnetic core, we find that we can increase this thickness, and that while the attractive force increases as the cube of the intensity, the thickness of the helix may with advantage be double the diameter of the core. The expression

$$\frac{\mathrm{E}^3t^3c}{\left(\frac{\lambda\pi ba(a+c)}{g^2}\right)^3}$$

is a maximum if c is variable when $a=2c$. On the other hand, there is no maximum when the forces are simply proportional to the intensities of the current.

Transmission of Power by Electricity.—A novel application of the electrical transmission of power has lately been made at the Trafalgar Collieries, Forest of Dean. The electrical arrangements were carried out by the Pyramid Electric Company, under the supervision of their managing director, Mr. A. Le Neve Foster. In this case an electric motor is used to drive a pump in the underground workings. The pump is employed for pumping the drainage water from some of the deep workings to the bottom of the shaft, from whence the ordinary steam pumps raise it to the surface. The total vertical lift of the electric pump is 115 feet whilst the length of pipes through which the water is forced is some 500 yards. A dynamo-machine is placed on the surface for generating the current for working the motor, and is connected to it by wires led down the shaft and along the workings, a distance of some 500 yards. Messrs. Brain are satisfied with the result of the undertaking, and propose still further to extend the utilization of electricity as a motive power.

ELECTRIC LIGHTING AND THE TRANSMISSION OF FORCE BY ELECTRICITY.

By DR. C. W. SIEMENS, F.R.S.

Amongst the practical questions that now chiefly occupy public attention are those of Electric Lighting, and of the transmission of force by electricity. I need hardly remind you that electric lighting, viewed as a physical experiment, has been known to us since the early part of the present century, and that many attempts have, from time to time, been made to promote its application. Two principal difficulties have stood in the way of its practical introduction, viz., the great cost of producing an electric current so long as chemical means had to be resorted to, and the mechanical difficulty of constructing electric lamps capable of sustaining, with steadiness, prolonged effects. The dynamo-machine, which enables us to convert mechanical into electrical force, purely and simply, has very effectually disposed of the former difficulty, inasmuch as a properly conceived and well constructed machine of this character converts more than ninety per cent. of the mechanical force imparted to it into electricity, ninety per cent. again of which may be reconverted into mechanical force at a moderate distance. The margin loss, therefore, does not exceed twenty per cent., excluding purely mechanical losses, and this is quite capable of being further reduced to some extent by improved modes of construction; but it results from these figures that no great step in advance can be looked for in this direction. The dynamo-machine presents the great advantage of simplicity over steam or other power-transmitting engines: it has but one working part, namely, a shaft, which, revolving in a pair of bearings, carries a coil or coils of wire admitting of perfect balancing. Frictional resistance is thus reduced to an absolute minimum, and no allowance has to be made for loss by condensation, or badly fitting pistons, stuffing boxes, or valves, or for the jerking action due to oscillating weights. The materials composing the machine, namely, soft iron and copper wire, undergo no deterioration or change by continuous working, and the depreciation of value is therefore a minimum, except where currents of exceptionally high potential are used, which appear to render the copper wire brittle.

The essential points to be attended to in the conception of the dynamo-machine, are the prevention of induced currents in the iron, and the placing of the wire in such position as to make the whole of it effective for the production of outward current. These principles, which have been clearly established by the labors of comparative few workers in applied science, admit of being carried out in an almost infinite variety of constructive forms, for each of which may be claimed some real or imaginary merits regarding questions of convenience or cost of production.

For many years after the principles involved in the construction of dynamo-machines had been been made known, little general interest was manifested in their favor, and few were the forms of construction offered for public use. The essential features involved in the dynamo-machine, the Siemens armature (1856), the Pacinotti ring (1861), and the self-exciting principle (1867), were published by their authors for the pure scientific interest attached to them, without being made subject matter of letters patent, which circumstance appears to have had the contrary effect of what might have been expected, in that it has retarded the introduction of this class of electrical machine, because no person or firm had a sufficient commercial interest to undertake the large expenditure which must necessarily be incurred in reducing a first conception into a practical shape. Great credit is due to Monsieur Gramme for taking the initiative in the practical introduction of dynamo-machines embodying those principles, but when five years ago I ventured to predict for the dynamo-electric current a great practical future, as a means of transmitting power to a distance, those views were still looked upon as more or less chimerical. A few

striking examples of what could be practically effected by the dynamo-electric current such as the illumination of the Place de l'Opera, Paris, the occasional exhibition of powerful arc lights, and their adoption for military and lighthouse purposes, but especially the gradual accomplishment of the much desired lamp by incandescence in vaacuum, gave rise to a somewhat sudden reversion of public feeling; and you may remember the scare at the Stock Exchange affecting the value of gas shares, which ensued in 1878, when the accomplishment of the sub-division of the electric light by incandescent wire was first announced, somewhat prematurely, through the Atlantic cable.

From this time forward electric lighting has been attracting more and more public attention, until the brilliant displays at the exhibition of Paris, and at the Crystal Palace last year, served to excite public interest, to an extraordinary degree. New companies for the purpose of introducing electric light and power have been announced almost daily, whose claims to public attention as investments were based in some cases upon only very slight modifications of well known forms of dynamo-machines, of arc regulators, or of incandescent carbon lights, the merits of which rested rather upon anticipations than upon any scientific or practical proof. These arrangements were supposed to be of such superlative merit that gas and other illuminants must soon be matters simply of history, and hence arose great speculative excitement. It should be borne in mind, however, that any great technical advance is necessarily the work of time and serious labor, and that when accomplished, it is generally found that so far from injuring existing industries, it calls additional ones into existence, to supply new demands, and thus give rise to an increase in the sum total of our resources. It is, therefore, reasonable to expect that side by side with the introduction of the new illuminant, gas lighting will go on improving and extending, although the advantage of electric light for many applications, such as the lighting of public halls and warehouses, of our drawing rooms and dining rooms, our passenger steamers, our docks and harbors, are so evident, that its advent may be looked upon as a matter of certainty.

Our Legislature has not been slow in recognizing the importance of the new illuminant. In 1879, a Select Committee in the House of Commons instituted a careful inquiry into its nature and probable cost, with a view to legislation, and the conclusions at which they arrived were, I consider, the best that could have been laid down. They advised that applications should be encourged tentatively by the granting of permissive bills, and this policy has given rise to the Electric Lighting Bill, 1882, promoted by Mr. Chamberlain, the President of the Board of Trade, regarding which much controversy has arisen. It could, indeed, hardly be expected that any act of legislation upon this subject could give universal satisfaction, because while there are many believers in gas who would gladly oppose any measure likely to favor the progress of the rival illuminant, and others who wish to see it monopolized, either by local authorities, or by large financial corporation, there are others again who would throw the doors open so wide as to enable almost all comers to interfere withe public thoroughfares, for the establishment of conducting wires, without let or hindrance.

The law as now established takes, I consider, a medium course between these diverging opinions, and, if properly interpreted, will protect, I believe, all legitimate interests, without impeding the healthy growth of establishments for the distribution of electric energy for lighting and for the transmission of power. Any firm or lighting company may, by application to the local authorities, obtain leave to place electric conductors below public thoroughfares, subject to such conditions as may be mutually agreed upon, the terms of such license being limited to seven years; or an application may be made to the Board of Trade for a provisional order to the same effect, which, when sanctioned by Parliament, secures a right of occupation for twenty-one years. The license offers the advantages of cheapness, and may be regarded as a purely tentative measure, to enable the firm or company to prove the value of their plant. If this be fairly established, the license would in all probability be affirmed, either by an engagement for its prolongation from time to time, or by a provisional order which would, in that

case, be obtained by joint application of the contractor and the local authority. At the time of expiration of the provisional order, a pre-emption of purchase is accorded to the local authority, against which it has been objected with much force by so competent an authority as Sir Frederick Bramwell, that the conditions of purchase laid down are not such as fairly to remunerate the contracting companies for their expenditure and risk, and that the power of purchase would inevitably induce the parochial bodies to become mere trading associations. But while admitting the undesirability of such consummation, I cannot help thinking that it was necessary to put some term to contracts entered into with speculative bodies at a time when the true value of electric energy, and the best conditions under which it should be applied, are still very imperfectly understood. The supply of electric energy, particularly in its application to transmission of power, is a ma ter simply of commercial demand and supply, which need not partake of the character of a large monopoly similar to gas and water supply, and which may therefore be safely left in the hands of individuals, or of local associations, subject to a certain control for the protection of public interest. At the termination of the period of the provisional order, the contract may be renewed upon such terms and conditions as may at that time appear just and reasonable to Parliament, under whose authority the Board of Trade will be empowered to effect such renewal.

Complaints appear almost daily in the public papers to the effect that townships refuse their assent to applications by electric light companies for provisional orders; but it may be surmised that many of these applications are of a more or less speculative character, the object being to secure monopolies for eventual use or sale, under which circumstances the authorities are clearly justified in withholding their assent; and no licenses or provisional orders should, indeed, be granted, I consider, unless the applicants can give assurance of being able and willing to carry out the work within a reasonable time. But there are technical questions involved which are not yet sufficiently well understood to admit of immediate operations upon a large scale.

Attention has been very properly called to the great divergence in the opinions expressed by scientific men regarding the area that each lighting district should comprise, the capital required to light such an area, and the amount of electric tension that should be allowed in the conductors. In the case of gas supply, the works are necessarily situated in the outskirts of the town, on account of the nuisance this manufacture occasions to the immediate neighborhood; and, therefore, gas supply must range over a large area. It would be possible, no doubt, to deal with electricity on a similar basis, to establish electrical mains in the shape of copper rods of great thickness, with branches diverging from it in all directions; but the question to be considered is, whether such an imitative course is desirable on account either of relative expense or of facility of working. My own opinion, based upon considerable practical experience and thought devoted to the subject, is decidedly adverse to such a plan. In my evidence before the Parliamentary Committee, I limited the desirable area of an electric district in densely populated towns to a quarter of a square mile, and estimated the cost of the necessary establishment of engines, dynamo-machines, and conductors, at 100,000*l*, while other witnesses held that areas from one to four square miles could be worked advantageously from one center, and at at a cost not exceeding materially the figure I had given. These discrepancies do not necessarily imply wide differences in the estimated cost of each machine or electric light, inasmuch as such estimates are necessarily based upon various assumptions regarding the number of houses and of public buildings comprised in such a district, and the amount of light to be apportioned to each, but I still maintain my preference for small districts.

By way of illustration, let us take the parish of St. James's, near at hand, a district not more densely populated than other equal areas within the metropolis, although comprising, perhaps, a greater number of public buildings. Its population, according to the preliminary report of the census taken on the 1st of April, 1881, was 29,865, it contains 3,018 inhabited houses, and its area is 784,000 square yards, or slightly above a quarter of a square mile.

To light a comfortable house of moderate dimensions in all its parts, to the exclusion of gas, oil, or candles, would require about 100 incandescent lights of from 15 to 18-candle power each, that being, for instance, the number of Swan lights employed by Sir William Thomson in lighting his house at Glasgow University. Eleven-horse power would be required to excite this number of incandescent lights, and at this rate the parish of St. James's would require $3.018 \times 11 =$ 33,200-horse power to work it. It may be fairly objected, however, that there are many houses in the parish much below the standard here referred to, but on the other hand, there are 600 of them with shops on the ground floor, involving larger requirements. Nor does this estimate provide for the large consumption of electric energy that would take place in lighting the eleven churches, eighteen club-houses, nine concert halls, three theatres, besides numerous hotels, restaurants, and lecture halls. A theatre of moderate dimensions, such as the Savoy Theatre, has been proved by experience to require 1,200 incandescent lights, representing an expenditure of 133 horse power; and about one-half that power would have to be set aside for each of the other public buildings here mentioned, constituting an aggregate of 2,926-horse power; nor does this general estimate comprise street lighting, and to light the six and a half miles of principal streets of the parish with electric light, would require per mile, thirty-five arc lights of 350 candle power each, or a total of 227 lights. This, taken at the rate of 0.8-horse power per light, represents a further requirement of 182-horse power, making a total of 3,108 horse power, for purposes independent of house lighting, being equivalent to one-horse power per inhabited house, and bringing the total requirements up to 109 lights = 12-horse power per house.

I do not, however, agree with those who expect that gas lighting will be entirely superseded, but have, on the contrary, always maintained that the electric light, while possessing great and peculiar advantages for lighting our principal rooms, halls, warehouses, &c., owing to its brilliancy, and more particularly to its non-interference with the healthful condition of the atmosphere, will leave ample room for the development of the former, which is susceptible of great improvement. and is likely to hold its own for the ordinary lighting up of our streets and dwellings.

Assuming, therefore, that the bulk of domestic lighting remains to the gas companies, and that the electric light is introduced into private houses, only, at the rate of, say twelve incandescent lights per house, the parish of St James's would have to be provided with electric energy sufficient to work $(9 + 12)$ 3,018 = 63,378 lights = 7,042-horse power effective; this is equal to about one-fourth the total lighting power required, taking into account that the total number of lights that have to be provided for a house are not all used at one and the same time. No allowance is made in this estimate for the transmission of power, which, in course of time, will form a very large application of electric energy; but considering that power will be required mostly in the day time, when light is not needed, a material increase in plant will not be necessary for that purpose.

In order to minimize the length and thickness of the electric conductor, it would be important to establish the source of power, as nearly as may be, in the centre of the parish, and the position that suggests itself to my mind is that of Golden-square. If the unoccupied area of this square, representing 2,500 square yards, was excavated to a depth of twenty-five feet, and then arched over so as to re-establish the present ground level, a suitable covered space would be provided for the boilers, engines, and dynamo machines, without causing obstruction or public annoyance; the only erection above the surface, would be the chimney, which, if made monumental in form, might be placed in the centre of the square, and be combined with shafts for ventilating the subterranean chamber, care being taken of course to avoid smoke by insuring perfect combustion of the fuel used. The cost of such a chamber, of engine-power, and of dynamo-machines, capable of converting that power into electric energy, I estimate at 140,000*l*. To this expense would have to be added that of providing and laying the conductors, together with the switches, current regulators, and arrangements for testing the insulation of the wire.

The cost and dimensions of the conductors would depend upon their length, and the electromotive force to be allowed. The latter would no doubt be limited, by the authorities, to the point at which contact of the two conductors with the human frame would not produce injurious effects, or say to 200 volts, except for street lighting, for which purpose a higher tension is admissible. In considering the proper size of conductor to be used in any given installation, two principal factors have to be taken into account; first, the charge for interest and depreciation on the original cost of a unit length of the conductor; and, secondly, the cost of the electrical energy lost through the resistance of a unit of length. The sum of these two, which may be regarded as the cost of electricity, is clearly least, as Sir William Thomson pointed out some time ago, when the two components are equal. This, then, is the principle on which the size of a conductor should be determined.

From the experience of large installations, I consider that electricity can, roughly speaking, be produced in London at a cost of about one shilling per 10,000 Ampère-Volts or Watts (746 Watts being equal to one horse-power) for an hour. Hence, assuming that each set of four incandescent lamps in series (such as Swan's, but for which may be substituted a smaller number of higher resistance and higher luminosity) requires 200 volts electromotive force, and 60 Watts for their efficient working, the total current required for 64,000 such lights is 19,200 ampères, and the cost of the electric energy lost by this current in passing through 1-100th of an ohm resistance, is 16*l.* per hour.

The resistance of a copper bar one quarter of a mile in length, and one square inch in section, is very nearly 1-100th of an ohm, and the weight is about $2\frac{1}{3}$ tons. Assuming, then, the price of insulated copper conductor at 90*l.* per ton, and the rate of interest and depreciation at $7\frac{1}{2}$ per cent., the charge per hour of the above conductor, when used eight hours per day, is $1\frac{1}{2}d.$ Hence, following the principle I have stated above, the proper size of conductor to use for an installation of the magnitude I have supposed, would be one of 48·29 inches section, or a round rod eight inches diameter.

If the mean distance of the lamps from the station be assumed as 350 yards, the weight of copper used in the complete system of conductors would be nearly 168 tons, and its cost 15,120*l.* To this must be added the cost of iron pipes, for carrying the conductors under ground, and of testing boxes, and labor in placing them. Four pipes of 10 inch diameter each, would have to proceed in different directions from the central station, each containing sixteen separate conductors of one inch diameter, and separately insulated, each of them supplying a subdistrict of 1000 lights. The total cost of establishing these conductors may be taken at 37,000*l.*, which brings up the total expenditure for central station and leads to 177,000*l.* I assume the conductors to be placed under ground, as I consider it quite inadmissible, both as regards permanency and public safety and convenience, to place them above ground, within the precincts of towns. With this expenditure, the parish of St. James's would be supplied with the electric light to the extent of about 25 per cent. of the total illuminating power required. To provide a larger percentage of electric energy would increase the cost of establishment proportionately; and that of conductors, nearly in the square ratio of the increase of the district, unless the loss of energy by resistance is allowed to augment instead.

It may surprise uninitiated persons to be told that to supply a single parish with electric energy necessitates copper conductors of a collective area equal to a rod of eight inches in diameter; and how, it may be asked, will it be possible under such conditions to transmit the energy of waterfalls to distances of twenty or thirty miles, as has been suggested? It must indeed be admitted that the transmission of electric energy of such potential (200 volts) as is admissible in private dwellings would involve conductors of impracticable dimensions, and in order to transmit electrical energy to such distances, it is necessary to resort in the first place to an electric current of high tension. By increasing the tension from 200 to 1,200 volts the conductors may be reduced to one-sixth their area, and if we are content to lose a larger proportion of the energy obtained cheaply from a waterfall, we may effect a still greater reduc-

tion. A current of such high potential could not be introduced into houses for lighting purposes, but it could be passed through the coils of a secondary dynamo-machine, to give motion to another primary machine, producing currents of low potential to be distributed for general consumption. Or secondary batteries may be used to effect the conversion of currents of high into those of low potential, whichever means may be found the cheaper in first cost, in maintenance, and most economical of energy. It may be advisable to have several such relays of energy for great distances, the result of which would be a reduction of the size and cost of conductor at the expense of final effect, and the policy of the electrical engineer will, in such cases, have to be governed by the relative cost of the conductor, and of the power at its original source. If secondary batteries should become more permanent in their action than they are at the present time, they may be largely resorted to by consumers, to receive a charge of electrical energy during the daytime, or the small hours of the night, when the central engine would otherwise be unemployed, and the advantage of resorting to these means will depend upon the relative first cost, and cost of working the secondary battery and the engine respectively. These questions are, however, outside the range of our present consideration.

The large aggregate of dwellings comprising the metropolis of London covers about seventy square miles, thirty of which may be taken to consist of parks, squares, and sparsely inhabited areas, which are not to be considered for our present purpose. The remaining forty square miles could be divided into say 140 districts, slightly exceeding a quarter of a square mile on the avarage, but containing each fully 3,000 houses, and a population similar to that of St. James's.

Assuming twenty of these districts to rank with the parish of St. James's (after deducting the 600 shops which I did not include in my estimate) as central districts, sixty to be residential districts, and sixty to be comparatively poor neighborhoods, and estimating the illuminating power required for these three classes in the proportion of 1 to $\frac{2}{3}$ to $\frac{1}{3}$, we should find that the total capital expenditure for supplying the metropolis with electric energy to the extent of 25 per cent. of the total lighting requirements would be—

$$20\times \quad 177{,}000 = 3{,}540{,}000l.$$
$$60\times\tfrac{2}{3}\times 177{,}000 = 7{,}080{,}000l.$$
$$60\times\tfrac{1}{3}\times 177{,}000 = 3{,}540{,}000l.$$
$$14{,}160{,}000l.$$

or say 14,000,000*l.*, without including lamps and internal fittings, and making an average capital expenditure of 100,000*l.* per district.

To extend the same system over the towns of Great Britain, Ireland and would absorb a capital exceeding certainly 64,000,000*l.*, to which must be added 16,000,000*l.*, for lamps and internal fittings, making a total capital expenditure of 80,000,000*l.* Some of us may live to see this capital realized, but to find such an amount of capital, and, what is more important, to find the manufacturing appliances to produce work representing this value of machinery and wire, must necessarily be the result of many years of technical developement. If, therefore, we see that electric companies apply for provisional orders to supply electric energy, not only for every town throughout the country, but also for the colonies, and for foreign parts, we are forced to the conclusion that their ambition is somewhat in excess of their power of performance; and that no provisional order should be granted except conditionally on the work being executed within a reasonable time, as without such a provision the powers granted may have the effect of retarding instead of advancing electric lighting, and of providing an undue encouragement to purely speculative operations.

The extension of a district beyond the quarter of a square mile limit, would necessitate an establishment of unwieldy dimensions, and the total cost of electric conductors per unit area would be materially increased; but independently of the consideration of cost, great public inconvenience would arise in consequence of the number and dimensions of the electric conductors, which could no longer be accommodated in narrow channels placed below the curb stones, but would necessitate the construction of costly subways —veritable *cava electrica*.

The amount of the working charges of

an establishment comprising the parish of St. James's would depend on the number of working hours in the day, and on the price of fuel per ton. Assuming the 64,000 lights to incandesce for six hours a day, the price of coal to be 20*s.* a ton, and the consumption 2lbs. per effective horse power per hour, the annual charge under this head, taking eight hours' firing, would amount to about 18,300*l.*, to which would have to be added for wages, repairs, and sundries, about 6,000*l.*, for interest with depreciation at seven-and-a-half per cent., 13,300*l.*, and for general management say, 3,400*l.*, making a total annual charge of 41,000*l.*, or at the rate of 12*s.* 9½*d.* per incandescent lamp per annum. To this has to be added the cost of renewal of lamps, which may be taken at 5*s.* per lamp of sixteen candles, lasting 1,200 hours, or to 9*s.* per annum, making a total of 21*s.* 9½*d.* per lamp for a year.

In comparing these results with the cost of gas-lighting, we shall find that it takes 5 cubic feet of gas, in a good argand burner, to produce the same luminous effect as one incandescent light of 16-candle power. In lighting such a burner every day for six hours on the average, we obtain an annual gas consumption of 10,950 cubic feet, the value of which, taken at the rate of 2*s.* 8*d.* per thousand, represents an annual charge of 29*s.*, showing that electric light by incandescenee, when carried out on a large scale, is decidedly cheaper than gas-lighting at present prices, and with the ordinary gas-burners.

On the other hand, the cost of establishing gas-works and mains of a capacity equal to 64,000 argand burners would involve an expenditure not exceeding 80,000*l.* as compared with 177,000*l.* in the case of electricity; and it is thus shown that although it is more costly to establish a given supply of illuminating power by electricity than gas, the former has the advantage as regards current cost of production.

It would not be safe, however, for the advocates of electric lighting to rely upon these figures as representing a permanent state of things. In calculating the cost of electric light, I have only allowed for depreciation and 5 per cent. interest upon capital expenditure, whereas gas companies are in the habit of dividing large dividends, and can afford to supply gas at a cheaper rate, by taking advantage of recent improvements in manufacturing operations, and of the ever-increasing value of their by-products, including tar, coke, and ammoniacal liquor. Burners have, moreover, been recently devised by which the luminous effect for a given expenditure of gas can be nearly doubled by purely mechanical arrangements, and the brilliancy of the light can be greatly improved.

On the other hand, electric lighting also may certainly be cheapened by resorting, to a greater extent than has been assumed, to arc lighting, which though less agreeable than the incandescent light for domestic purposes, can be produced at less than half the cost, and deserves on that account the preference for street lighting, and for large halls, in combination with incandescent lights. Lamps by incandescence may be produced hereafter at a lower cost, and of a more enduring character.

Considering the increasing public demand for improved illumination, it is not unreasonable to expect that the introduction of the electric light to the full extent here contemplated, would go hand in hand with an increasing consumption of gas for illuminating and for heating purposes, and the neck-to-neck competition between the representatives of the two systems of illumination, which is likely to ensue, cannot fail to improve the quality, and to cheapen the supply of both, a competition which the consuming public can afford to watch with complacent self-satisfaction. Electricity must win the day, as the light of luxury; but gas will, at the same time, find an ever-increasing application for the more humble purposes of diffusing light.

In my address to the British Association I dwelt upon the capabilities and prospects of gas, both as an illuminant and as a heating agent, and I do not think that I was over-sanguine in predicting for this combustible a future exceeding all present anticipations.

I also called attention to the advantages of gas as a heating agent, showing that if supplied specially for the purpose, it would become not only the most convenient, but by far the cheapest form of fuel that can be supplied to our towns. Such a general supply of heating separ-

ately from illuminating gas, by collecting the two gases into separate holders, during the process of distillation, would have the beneficial effects—

1. Of giving to lighting gas a higher illuminating power.

2. Of relieving our towns of their most objectionable traffic—that in coal and ashes.

3. Of effecting the perfect cure of that bugbear of our winter existence—the smoke nuisance.

4. Of largely increasing the production of those valuable by-products, tar, coke, and ammonia, the annual value of which already exceeds by nearly 3,000,000*l.* that of the coal consumed in the gas-works.

The late exhibitions have been beneficial in arousing public interest in favor of smoke abatement, and it is satisfactory to find that many persons, without being compelled to do so, are now introducing perfectly smokeless arrangements for their domestic and kitchen fires.

The Society of Arts, which for more than 100 years has given its attention to important questions regarding public health, comfort, and instruction, would, in my opinion, be the proper body to examine thoroughly into the question of the supply and economical application of gas and electricity for the purposes of lighting, of power of production, and of heating. They would thus pave the way to such legislative reform as may be necessary to facilitate the introduction of a national system.

If I can be instrumental in engaging the interest of the Society in these important questions, especially that of smoke prevention, I shall vacate this chair next year with the pleasing consciousness that my term of office has not been devoid of a practical result.

THE REGULATION OF RIVERS AND WATERWAYS, WITH A VIEW TO THE PREVENTION OF FLOODS.

By GUSTAV RITTER VON WEX Privy-Councillor to His Imperial Majesty the Emperor of Austria.

Translated from the German for the Institution of Civil Engineers by WM. ATKINSON BELL.

THERE is such an infinite variety in the characteristic features of each waterway, as regards the configuration and geological structure of the country it permeates—whether its course lie in the interior of some great continent, and it empties itself into some other river, or into an arm of the sea, more or less affected by the ebb and flow of the tide—and also as to the extent and nature of the floods and their effect, that an exhaustive description of the regulating works applicable to each case would fill a folio volume. This memoir will therefore be limited to a brief general summary of the first principles requisite to the successful regulation of intractable rivers.

I.—REGULATION OF THE NON-TIDAL PORTION OF RIVERS.

In every case, first of all the upper course of the river must be dealt with separately, and then the lower portion of it, together with its mouth, whether it empties itself into an estuary or into the open sea.

From long experience it has been ascertained that every river or stream, following its natural course through wide tracts of level country, invariably, if the banks consist of deposits of earth or gravel, attacks them, the lighter particles being carried away, the heavier being deposited in the bed of the stream, so that in course of time its width increases while its depth decreases, and at the same time islands, sand-banks, bends, creeks, and by-channels are formed.

In rivers thus left to nature the fall, mean velocity, and force of the current are continually decreasing while the river-bed is rising; this naturally raises the general water-level relatively to the adjoining country, and exposes it to frequent inundation, the effects of which are disastrous floods and the formation of innumerable branches and by-channels which intersect the whole country, flooding and swamping it at every rise

of the river, and rendering it in time unfit for habitation by either man or beast. Instances of the kind are at the present time to be met with in many parts of the world, notably in Asia, Africa, and America.

In order to deal effectually with such cases, namely, to abate the floods, and to prevent the disasters accompanying them, as well as the ultimate formation of trackless swamps, the following procedure is recommended:

(1.) A new channel following the course of the valley should be carefully laid down by the superintending engineer, either in a direct line or with easy bends, and when excavated, the entire body of water should be admitted into this new channel, the old bed and all by-channels being filled up.

(2.) Having carefully determined—

(*a*) The discharge per second at low, mean, and high water-level of a cross-section of the river, either immediately above or immediately below the portion to be regulated, and

(*b*) The increased fall which the new channel will afford; then the sectional area of the new bed must be fixed, according to approved hydraulic formulas, so as to allow of the passage of either an ordinary or an extraordinary volume of water.

(3.) The water having been admitted, the next thing is to protect the banks by random rubble or by stone pitching in order to prevent the action of the current injuring them, or forming bends or creeks.

(4.) After the completion of the above, the old river-bed and by-channels should be filled up, the land thus reclaimed should by degrees be brought under cultivation; in the same manner the marshy tracts exposed hitherto to innudation, and fertilized by the deposit therefrom, should be raised by a coating of rich soil.

(5.) If exceptionally high floods still overflow the banks and inflict loss and damage to the freshly cultivated valley, dikes at suitable distances apart will be necessary to confine such floods, and enable them to flow off gradually without causing damage.

From forty-eight years' observation and experience of extensive works undertaken for the improvement of rivers, the author can confidently affirm that by careful attention to the points above recommended, even the most tortuous rivers and the swampiest valleys have, generally within a few years, but in some cases only after many years, yielded the most satisfactory results, as for instance—

(*a*) The increase of fall due to the more uniform section and more direct course, and the concentration and confinement of the stream within a single channel provided with firm banks, considerably increase the force and velocity of the current, which tend to deepen the channel, and to carry away the material thus scoured out as well as that brought down from above.

(*b*) By lowering the bed of the river, in some cases to the extent of from 3 to 6 feet, the general water-level, both of the river, and of the ground springs in the neighborhood, is proportionately lowered, so that the adjoining country is less liable to inundation, and the swamps are more easily drained and brought under cultivation.

(*c*) The velocity being accelerated in the new channel, as shown above (*a*), floods pass off more rapidly and do not rise so high, consequently the low country is seldom or ever under water, or at any rate not to the same extent as before. If, however, these lesser and lower floods are to be entirely prevented, dikes parallel to the course of the river must be added.

(*d*) In rivers exposed to the action of frost, floating ice is apt to accumulate in the unregulated portions of its course, especially at sharp bends, and on shallows and sand-banks, occasionally to such an extent as entirely to obstruct the flow of the stream, and to raise the water in the river to such a height that it overflows the banks, inundates the neighboring country, and spreads ruin far and wide.

When once a river has been regulated this cannot take place, as there would then be nothing to hinder the free passage of floating ice, and should a temporary stoppage occur, the concentrated force of the current would soon overcome every obstacle, by raising the blocks, and bearing them away without causing any flood.

(*e*) It is a matter of general experi-

ence that even in a deep river following a winding course and dividing into numerous branches, navigation is often obstructed to such an extent that the river becomes all but impassable, yet when the same river has been regulated, a direct channel is provided, facilitating traffic and commerce, and increasing the prosperity of the country already improved by drainage and cultivation.

(*f*) On the banks of many rivers left to Nature but a scanty population exists, invariably affected and often decimated by epidemics, and generally exhibiting an imperfect physical and mental development. After regulation, and by draining and cultivating the adjoining country, these evils disappear, the inhabitants improve in health, strength and intelligence, the population increases, new villages spring up, and prosperity reigns where before disease and poverty were rife. The Government earns the hearty thanks of all thus benefited, and is at the same time fully recouped the capital laid out on the works by the increased revenues derived from the improved condition of the country.

Where such an improvement of the waterway has been rationally executed, in accordance with the particular nature and requirements of the locality, most, if not all, of the above advantages have been secured; as may be proved by numerous instances of works of the kind executed years back in France, Germany, Austria, Switzerland, and Italy. Moreover, the fact that the Chambers of Deputies of these States have, during the last few years, repeatedly devoted hundreds of millions of florins to the completion of works already begun, and to new undertakings of the same kind, is a proof that the importance and advantage of such improvements are fully recognized.

As a complete description of even the most important works of this kind would far exceed the limits of a short paper, the author must confine himself to a brief review of those successfully accomplished on the Rhine and the Danube.

The Rhine, between Basle and Mannheim, has for centuries followed a tortuous course, abounding in sharp bends and dividing into many branches, through a valley between 5,000 and 6,000 meters broad. Having further repeatedly shifted its course, the whole valley became cut up by old channels; to a considerable extent, too, its natural fall was lost, owing to its sinuous course, and consequently the rate and force of the currents were so much diminished, that deposit accumulated everywhere, raising the bed of the river and mean water-level to such a degree that the adjoining country was little better than a swamp. The bed of the Rhine being thus elevated, and its course so irregular, the flood-water could not flow off rapidly enough, but spread abroad, inundating the neighborhood, and destroying whole villages and townships.

The riverside communities had in all ages attempted, by dams and other protective works, to abate these evils, but with little success, as, owing to the winding course of the river, the floods confined at one point escaped at another, and took their defences in reverse. This deplorable condition of the Rhine valley continued until the commencement of the present century, when the population, already greatly reduced by poverty and disease, was daily decreasing owing to emigration to America.

Colonel Tulla, of the Engineers, an eminent authority on hydraulics at that time, by repeated and unremitting exertions, induced the Government in 1817 to undertake a thorough survey of the entire Rhine valley. Upon that survey was based the project for the radical regulation of the Rhine bed, which was approved and ratified by treaty between the border States of France, Bavaria, and the Grand Duchies of Baden and Hesse, and according to which the regulation of the Rhine was carried out during the years 1819 to 1863.

The work consisted in regulating the course of the river and making it more direct. This necessitated the excavation of twenty-three considerable cuts, which reduced the distance by river between Mannheim and Basle from 252 to 169 kilometers, and increased the fall 30 per cent. Further, the stream was confined to a uniform channel of suitable section, both banks were substantially protected, the old river-bed and all branches were filled in, and the land thus reclaimed was for the most part brought under cultivation.

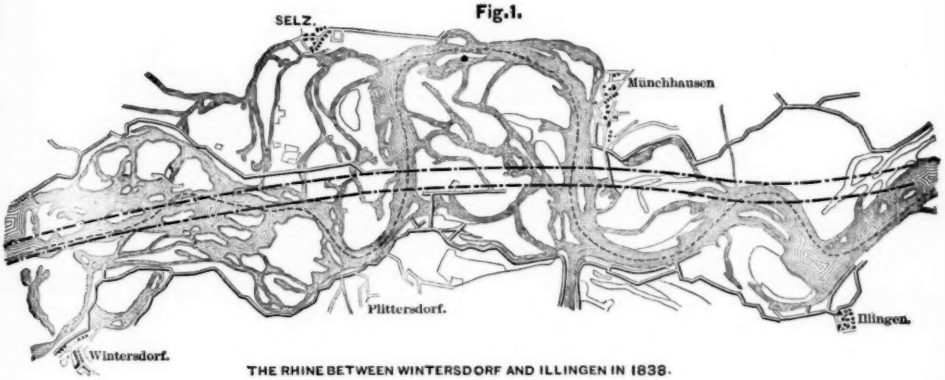
The above-mentioned regulation of the Rhine may be considered one of the most extensive and interesting undertakings of the kind ever attempted in Europe, the scope of which can scarcely be fully appreciated without reference to the exhaustive description and numerous plans issued by the Public Works Department of the Grand Duchy of Baden at Carlsruhe. By these it is proved that the following advantages have been secured, viz.:

(*a*) The river has undeviatingly followed the new course provided for it, has deepened its bed to the extent of two meters in some places, and lowered the mean water-level proportionately. Flood-water also has been passed more quickly.

(*b*) The general water-level being thus reduced in height, extensive tracts of swampy ground have been laid dry and converted into fertile arable land. Further, more than 20,000 hectares of old river bed water-holes and sand-banks have been reclaimed, and brought in a great measure under cultivation; and, lastly, the low-lying tracts are now no longer exposed to inundations.

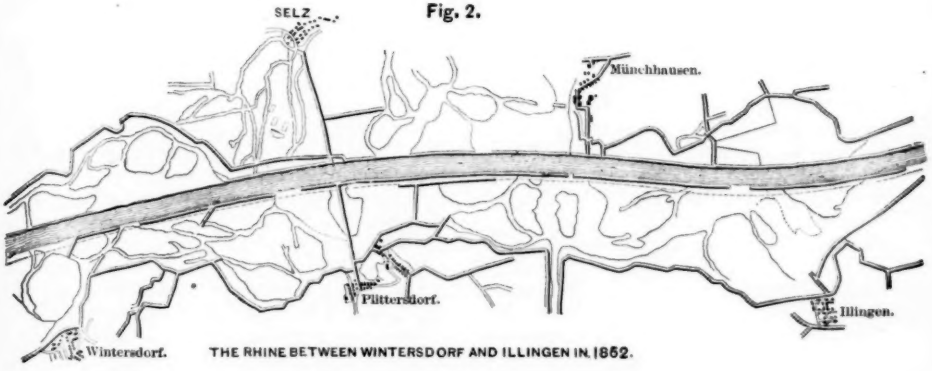
(*c*) The sanitary condition of the Rhine valley has visibly improved, and the general prosperity of the inhabitants materially increased.

Fig. 1.



THE RHINE BETWEEN WINTERSDORF AND ILLINGEN IN 1838.

Fig. 2.



THE RHINE BETWEEN WINTERSDORF AND ILLINGEN IN 1862.

(*d*) According to the concurrent reports of experts, Government officials, and local authorities, the benefits derived from the regulation of the Rhine are so considerable that the capital laid out has been amply repaid. Wherefore in grateful recognition of the emin-

ent services rendered by Colonel Tulla, the original promoter of the scheme, a statue has been erected in his honor at Maxau on the banks of the river.

In one respect only has the regulation of the river not fulfilled the expectations of its promoters, viz., the extensive sandbanks formed at the confluence of its tributaries have rendered intercommunication with them both difficult and dangerous, because these feeders to the main stream enter it across bars little more than 0.60 or 1.50 meter below low-water level.

Had the hydraulic engineers in 1817 correctly determined the minium discharge of the Rhine, and at the same time anticipated a probable decrease of the same, they would have diminished the waterway, and thereby considerably reduced the deposit and formation of such sandbanks, so that the channels of communication between the Rhine and its tributaries would have remained more open to navigation.

As a second example of the satisfactory results of the regulation of an important river, the works executed on the Danube, near Vienna, during the years 1869-1881, may be cited. These were described in a paper communicated to the Austrian Society of Engineers and Architects, and published in their journal for 1880.

The course of the Danube at Vienna was by no means so irregular as that of the Rhine. Owing, however, to three sharp bends in the immediate vicinity of the city, and to the width of the river and numerous shoals, vast masses of ice accumulated, almost every severe winter, causing the suburbs to be flooded, and leading to great damage. Moreover, the navigation was much impeded by shoals and shallows, and was exceedingly dangerous between the narrow bays of the wooden bridges. It was impossible to build masonry bridges, and there were no commodious landing-places, as the only place adapted to the purpose was the long sweep of the river at Florisdorff, some distance from Vienna and on the opposite side.

This state of matters was very injurious both to the health and to the commercial prosperity of the city, yet it was not until many years had been consumed in negotiations between the Government, the Landtag, and the Vienna Municipality, that at length a well-considered project for regulating 23 kilometers of the course of the river, from Nussdorf to Fishamend, was sanctioned and commenced with a capital of 32 million florins. The results of this work, already apparent, are as follows:

1. Although the cut (6,440 meters in length) between Nussdorf and Fishamend was only opened in 1876, and the other works were still incomplete, yet the extraordinary quantity of ice that came down the river during the severe winters of 1876 and 1880, as well as the high floods in August, 1880, passed Vienna without causing the slightest damage. It may, therefore, be taken for certain that these dangers to the city have been effectually removed. It is beyond doubt that had the regulation works not been constructed, the huge banks of ice piled up on both sides of the river a short distance below Vienna, would have spread over the low-lying suburbs, as was the case in 1830, and have caused incalculable damage and loss of life and property.

2. Since the low-lying suburbs have been protected against the floods and ice, and the general water levels of the country and of the river have been lowered about 0.50 meter, the epidemics formerly so prevalent have all but disappeared, the sanitary condition of the population has improved, and the death rate has been considerably lowered.

3. The river is now 1,890 meters nearer Vienna, and in order to extend the city in that direction a site for a new suburb has been supplied, by filling up the old river-bed, &c., equal to 267 hectares, available for building purposes, after deducting the space to be occupied by projected streets, squares, and gardens.

By diverting the river it has also been possible to remove the old pile-bridges and to replace them by five handsome stone structures, two being for ordinary and three for railway traffic, the details of which are given in the report issued on the occasion of the public opening of the new channel for navigation on the 30th May, 1876.

The course of the river at present affords a navigable channel of a uniform section and easy bends, with a depth of 3 meters at the lowest water level, so that the navigation is never interrupted.

Three extensive sandbanks have been

formed on each side of the river, but offer no obstacle to navigation, as they are everywhere at least 0.60 to 1 meter below the lowest water level, and they do not extend across the river; this is attributable to the fact that, profiting by experience gained on the Rhine, the width originally determined on, viz., 1,000 feet, was reduced to 900 feet.

5. Along the right bank of the new channel, besides public landing stages, ten quays 1,067 meters long have been constructed, and 13,276 meters of frontage secured, on which numerous warehouses have been built. These have been put into direct communication with all the railway stations in Vienna by the new line along the banks of the Danube, so that all goods arriving at or leaving Vienna may be transferred from the shipping to the railway, or *vice versa*, with the greatest facility, expedition, and economy. A roomy winter harbor is also provided for unloading the river craft during floods and during the breaking up of the ice.

In consequence of the above improvements the traffic and commerce on the Danube has quadrupled since the river has been regulated. It is also evident that all the results anticipated have been fully realized, and so many substantial benefits, both to the city and to the country, generally secured, that the immense capital of 32 million florins has been well expended and richly repaid. Mr. James Abernethy, Past-President Inst. C. E., greatly contributed to its success by strongly recommending the plan since executed, and assisting in the elaboration of the entire scheme, when summoned to Vienna in 1867.

The manifest advantages secured by the regulation of this portion of the Danube have induced the Government, Landtag, and Municipality to authorize forthwith the regulation of further portions of the river, namely, from the confluence of the Isper to Nussdorf, and from Fishamend to Theben, on the Hungarian frontier, a length of 157 kilometers, at an estimated cost of 24 million florins.

The next section, from Pressburg to Gönyö, 80 kilometers in length, is quite as irregular as the course of the Rhine between Basle and Mannheim was before 1817. Yet the Hungarians object to its regulation because they are jealous of any increase of traffic between the lower Danube and Vienna, and are anxious to attract it to Budapest. But lately the Hungarian Government has discovered, that the devastation caused to the adjoining country, by floods in this unregulated portion of the river, is on the increase, and that the present difficult navigation has not so much affected Vienna as their own export trade to the westward. It has therefore determined to proceed with the regulation of that portion of the Danube between Pressburg and Gönyö within their jurisdiction, at an estimated cost of about 20 million florins.

Advantage may be taken of this opportunity to refer briefly to the circumstances which prevented the regulation of the Theiss in Hungary being productive of any permanent improvement, and in fact rendered it a complete failure. It is beyond a doubt that the depressed Hungarian basin was at one time occupied by a vast inland sea, which gradually drained away after the Danube had forced a passage through the mountains between Moldova and Orsova.

The river Theiss receives the entire drainage from the Carpathian mountains, and after a most tortuous course through the level plains of the above basin, delivers it into the Danube. Throughout its whole course below Szolnok it falls at the rate of only 1 in 46,000, and latterly of 1 in 60,000; whereas in its upper course, between Tokay and Csáp, the fall is at the rate of 1 in 19,000 to 1 in 5,200. The wealthy landowners on this upper portion of the river have protected their own property from the effects of floods by providing a direct course for the Theiss, and confining it to that course by a continuous system of dikes. The natural fall has therefore been increased, and consequently it now bursts with redoubled force into the lower reaches, where the fall is very slight. Besides, in the lower portion of the river, little has been done to provide a more direct channel, and the dikes thrown up on either side have been constructed in such a haphazard manner that in some places they are actually at right angles to the course of the river, besides being at very unequal distances apart, ranging from 400 to 1,400 meters.

Lastly, no increase to the cross-section

of the channel has been attempted, though repeatedly recommended by the most eminent hydraulic engineers; nothing has been done to improve the course of the river Maros, which flows into the Theiss at right angles to the latter, nor has its junction with the Danube been regulated. It is but natural that the works undertaken on the Theiss, and executed in direct opposition to the first principles of hydraulics, should have proved most unsatisfactory; that floods should have been intensified, the dikes breached, and wide fertile tracts inundated and laid waste—a calamity that, on the 12th March, 1879, befel the populous commercial city of Szegedin.

These unsatisfactory results should in no way shake confidence in the undeniable and proved success of many rationally executed works of the kind in Europe and elsewhere, but rather serve as an example and a warning that improvements conducted regardless of true scientific principles can only end in that way.

The depth of water in almost every river having sensibly decreased within the last ten years, to the serious obstruction of navigation, whereas at some seasons the floods have risen even higher than before, some German engineers have given it as their opinion that this is owing to the regulation of rivers, as they affirm that the ordinary flow passes off more rapidly when confined to a channel of a regular section and direct course; for the same reason the floods also descend more quickly from the high ground and rise to a greater height. Arguing from the above hypothesis, these engineers maintain that the execution of regulating works is disadvantageous, and propose instead the construction of capacious reservoirs to impound the surplus rainfall, so as to prevent floods and at the same time to regulate and maintain an equable flow of water at all seasons.

The author cannot assent to this theory. In all the papers published by him from 1873 until 1879, he has clearly shown that during the last ten years the supply of water derived from natural springs has perceptibly decreased, and consequently the level in rivers, &c., has been lowered. This would have been still further the case had the rivers remained unregulated, because the same quantity of water spread over a wide irregular bed, and distributed in many branches, would expose a far larger surface to the atmosphere, and consequently more water would be lost by evaporation.

Floods at the present day occur in the regulated channels, rising even higher than formerly, in spite of the rapid overflow, and for this reason, that in consequence of the clearance of forests, especially on mountain tracts, rain-storms are more frequent; further, because the reduced total rainfall penetrates nowadays less into the ground, and flows off from the treeless slopes of the mountains and country, now intersected by numerous ditches and drains, into the rivers much more rapidly and in greater quantities, which accounts for these heavy floods.

The extraordinary advantages of constructing reservoirs in the drainage area of rivers was recognized by the Chinese four thousand three hundred years ago, and even at that time many reservoirs several square miles in extent were constructed in connection with the regulation of their large rivers.

In the present day, when every square yard of ground, even in mountainous districts, is private property, and obtainable only at almost prohibitive prices, the construction of huge reservoirs in the drainage area of rivers would be both difficult and expensive. Besides, such reservoirs could be of little or no use in connection with an unregulated river, consequently its regulation would in the end be obligatory.

Latterly, numerous ideas and projects for regulating and making navigable natural river courses, and at the same time preventing floods, have been propounded. But regarding them as impracticable, and as unlikely to lead to any favorable results, it is needless to enter into a discussion of their merits.

II.—Regulation of Rivers emptying into the Sea, or into an Arm of the Sea.

The irregular course of rivers flowing into the sea is due principally to the facts, that in the lower part of their course the inclination of the bed is inconsiderable, and that frequently bars are formed at the mouth by the joint action of the river and the sea. The bars also prevent the egress of the land water, driving it back and flooding the

adjoining country, until the ebb of the tide permits the river to flow again and find its way into the sea, by the numerous channels it has forced through the delta at its mouth.

The regulation of rivers flowing into the sea is a much more difficult and a more costly undertaking than that hitherto treated of; for the volume of water to be dealt with is greater, and the loose and yielding nature of the deposit forming the banks and bed of the river, as well as the violence of the storms and force of the waves, render it necessary that the protection of the banks and all structures should have solid foundations and be executed in the most substantial manner.

The ordinary works necessary to the perfect regulation of such a river consist of—

(1) Rendering the course as nearly straight as possible in order to increase the fall.

(2) Enclosing the river near its outfall by means of dikes, and continuing the same beyond the bar and as far out to sea as possible, that the force of the current may be sufficient to carry the sand and mud to such a depth that the action of the sea may carry it away and prevent the formation of a bar at the mouth.

If the adjoining country is to be effectually protected from inundation produced by the action of the tide, it will be necessary on either bank, beyond the limit reached by the tide, to construct dikes at such distances and in such a manner that at ebb tide the force of the river may be sufficient to carry out to sea the silt, &c., deposited during the flow of the tide; when the river is thus confined, then the low country beyond the dikes may be drained and brought under cultivation.

It is not proposed to discuss the works executed for regulating rivers falling into the sea. It may suffice to mention the celebrated works at the Sulina mouth of the Danube, those at present in course of completion at the mouth of the Mississippi; and, lastly the successful drainage and cultivation of the deadly swamps of Tuscany and the lowlands of Holland, lying 5 feet below mean sea-level.

The vast experience gained in Europe in works of this description justifies the opinion that the science of hydraulics has, thanks to the observations carefully recorded for many years, arrived at such perfection that every practical hydraulic engineer is in a position to project and execute works calculated to rectify a bad waterway, to drain a swampy tract, and to prevent the recurrence of floods and their dreadful consequences.

Lastly, a few remarks may be permitted upon the papers by Mr. Wheeler and Mr. Jacob on the Conservancy of Rivers, dealing respectfully with the Eastern Midland District of England, and with the Valley of the Irwell. From these communications it would appear that the irregularities in the course of the four rivers draining the Eastern Midland counties, and of the Irwell, were not due so much to the configuration of the country or to natural causes, but

(1) Chiefly to the independent way in which, on their own section, each proprietor carried out the improvements he judged best, without any regard to the very first principles of hydraulics, and utterly regardless of their effect on the course of the river above or below him.

(2) To the fact that the weirs, sluices, and other works, were built too high and too narrow, thus facilitating the formation of sandbanks and the silting up of the river-bed, and so hemming in the ordinary freshets as to force the water over the banks and flood the adjoining country.

(3) The scouring and deepening the bed of the river by the action of the tide was rendered impossible by the numerous weirs, sluices, and other obstructions connected with the supply of water of the mills, as well as by the bad management of the same, so that not unfrequently the floods were permitted to lie on the land for weeks together, rendering it little better than a swamp.

According to Mr. Jacob, all the slag, refuse, and even the earth excavated for the foundations of smelting furnaces, factories, and dwelling-houses, in the immediate vicinity of the Irwell, were shot into the river or on to its banks, to be swept away by floods into its bed; so that it is scarcely to be wondered at, that rivers so treated should at length become so obstructed as to cause frequent and destructive floods.

The evils thus produced by ill-designed

structures and by the bad management of those in charge of them were further augmented by two circumstances:

(1) That the bar at the mouth of the river raised the ordinary water-level, and dammed up and forced back the flood waters, so that the country around was continually exposed to inundation.

(2) In consequence of improved cultivation and better drainage the rainfall in the upper basin of the river flowed off in greater quantities and more rapidly than formerly, while the section of waterway, and the weirs and sluices on it had not been proportionately increased; this raised the height of the floods and increased the area subject to inundation.

From the minute particulars given by Messrs. Wheeler and Jacob, the author feels himself in a position to express a general agreement with the suggestions made by them for regulating the rivers and remedying the evils complained of, because they evidently coincide with the principles applied with success to many works of a similar kind in Europe and elsewhere. On the strength of long practical experience in this branch of the profession, he would beg to be allowed to make the following remarks:

(*a*) In the first place, the discharge per second at important points on each of these five rivers during high floods should be carefully ascertained, so that a suitable cross-section for the channel and for the sluices might be fixed.

(*b*) The masonry weirs across the bed of the rivers should undoubtedly be removed, as at present they tend to augment the floods and to facilitate the silting up of the bed, and if allowed to remain, would render a complete regulation of the river either impossible or nearly useless.

(*c*) The sills and floors of all existing sluices and locks, as well as of those to be constructed in place of the present weirs across the bed of the river, should be laid several feet below the present river-bed, in order that, after the regulation, the anticipated lowering of it may meet with no obstacles.

If a thorough regulation and removal of the present ill-designed regulating works be carried out, it is certain that the serious floods complained of will cease, the inundation of the adjoining land disappear, the greater portion of these rivers be rendered navigable, agriculture flourish, and the health and well-being of the population improve, the mortality decrease, and the capital expended thereon give an abundant return.

From the concluding remarks of the above gentlemen the author gathers that none of the improvements advocated by them can be put in hand, until Parliament has passed an Act for the better regulation and control of all works erected by private enterprise, and for a due inspection of the same. The institution of Civil Engineers might, therefore, render immense service to the inhabitants of many river-basins were it to exert its influence in promoting such an Act of Parliament, and aiding to the best of its ability river improvements of this description.

Steel Marine Boilers.—The progress which has been made during the past few years in the employment of steel for marine boiler construction is strikingly exemplified by some interesting data with which we have been favored by the Wallsend Slipway and Engineering Company of Newcastle-on-Tyne. In the three years —1878, 1879, 1880—that firm constructed seventy five marine boilers of the aggregate weight of 1808 tons and of these sixteen, or say 21 33 per cent., were of steel. In 1881 they constructed forty boilers, weighing collectively 1089 tons, and of twenty five, or say 62½ per cent., were of steel, while during the current year the number of boilers made by the firm has been fifty-six, of the gross weight of 1360 tons, and of these no less than forty-seven, or 84 per cent. of the whole, were of steel. It will thus be seen that as far as the Wallsend Slipway and Engineering Company is concerned the relative positions of steel and iron, as materials for boiler construction, have been entirely reversed during the past two years. It will be remembered that in April, 1878, Mr. William Boyd, of the above-named firm, read before the Institution of Mechanical Engineers a valuable paper on "Experiments Relative to Steel Boilers," in which the value of steel, as a material for boilers, was most thoroughly discussed.

INFLUENCE OF THE EARTH'S ROTATION ON DERAILMENTS.

By RD. RANDOLPH, C. E.

Written for VAN NOSTRAND'S ENGINEERING MAGAZINE.

IT has often been stated in scientific publications that upon railways lying north and south there was a tendency on the part of the cars to quit the track and to pass to the eastern side when moving northward, and to the western side when moving southward. It is also stated that this tendency is so strong that a large majority of the derailments on tracks so located have been towards the side, indicated by this theory. It may be interesting to analyze this influence of the earth's rotation upon bodies moving upon its surface, and to reduce the force at different latitudes to its equivalent in pounds. Such an examination will show that the pretended observations of derailments could not have been made, and that the force in question is too slight to have any appreciable effect upon them.

When a locomotive runs upon a track in the form of a circular curve, it is constantly and uniformly forced from the direction of its last impulse by the projecting rim of the wheel coming in contact with the rail, each part of which is constantly deflecting from the direction of its adjoining part in the same horizontal plane; the parts being considered infinitely short in theory and indefinitely short in practice. There is thus a constant tendency to depart from the curved line and to follow the direction of the last infinitely short piece of track; which is that of the tangent at the point of departure. But the curvature of the rail, in order to constantly deflect the moving body from its former course, is only necessary when it is fixed to a stationary plane. If, however, the horizontal plane is itself rotating about a perpendicular axis the rail fixed to the plain may be straight and yet have exactly the same effect upon the moving body as if the plane were stationary and the rail curved; it being only necessary that the moving body be forced to deflect from its former course at every point at the same rate on either the curved or straight line. In one case only that part of the line which forces the moving body makes the required deflection. In the other the whole line makes the required deflection. which does not affect the lateral force communicated to the body.

If a straight line in a plane deflects about a fixed point at one end until it returns to its first position, it will have described an angle of 360 degrees, and every small part of that line will have also deflected the same amount about its own terminal point. The same radius of the moon is always pointed towards the earth and may be considered a part of the radius which describes the moon's orbit about the earth; yet the moon makes one revolution about its own axis in the same time it makes one rotation in its orbit. An observer outside of the orbit would see all the points of its surface in succession. And if there were a succession of moons located along the radius of the orbit, their radii constituting parts of this radius and moving in the same periods, each lunar radius would make the same deflection as the orbital radius in the same time. Therefore if a body should move along such a deflecting line, it would constantly meet a portion of the line having a different direction from that of the motion which had just been imparted to the body, just as if it were running over a curve deflecting the same amount in the same time and distance.

The amount of deflection of a line about a fixed point is measured by the sum of the infinitely small angles described by the deflecting line; or, when the length of the line is constant, by the sum of the infinitely small arcs described by the end of the line; and in general, by the sum of these arcs divided by the length of the line. If the deflecting line describes a plane, when it returns to its first position, it has described 360 degrees. If the line of the same

length describes a cone, the number of degrees deflected will have the same proportion to 360 as the circumference of its base has to the circumference of the circular plane. If the line describing the cone is of a different length from that describing the plane, the amount of deflection in each case will be in proportion to the circumferences divided by their respective describing lines. When a cylinder is described the divisor becomes infinite, and consequently the amount of deflection reduces to zero.

Now a tangent to the earth's circumference intersecting its axis will, at the pole, describe a plane; approaching the equator, it will describe a series of cones increasing in acuteness until at the equator, where such tangent becomes infinite, the cylinder and the zero of deflection is reached. The cotangent of the degree of latitude of a point on the earth's surface is this describing line, and the cosine of the latitude is the radius of the base of the cone. The circumference of the base of the cone described by the cotangent at any point of latitude is cos. $\times 2\pi$. If the same cotangent had described a plane instead of a cone the circumference of the circle would be cot. $\times 2\pi$. Therefore if cot. $\times 2\pi$ subtends 360 degrees, cos. $\times 2\pi$ will subtend

$$\frac{\text{cos.} \times 2\pi \times 360^\circ}{\text{cot.} \times 2\pi} = 360^\circ \times \frac{\text{cos.}}{\text{cot.}}$$

which is the amount of deflection of a tangent to the earth's surface in twenty-four hours, or one rotation.

If D is the distance on a straight track over which the velocity of the locomotive would take it in twenty-four hours, and this distance should be laid out on an immovable plain in the form of a circular arc containing the same amount of deflection which a tangent to the sphere at a certain degree of latitude would perform by the earth's rotation in twenty-four hours, it will subtend an angle of $360^\circ \times \frac{\text{cos.}}{\text{cot.}}$; and this arc will have the same proportion to the whole circle as cos. is to cot. Let R be the radius of the arc D; then the circle will be R $\times 2\pi$; which having the same proportion as cos. to cot.

$$\frac{\text{D} \times \text{cot.}}{\text{cos.}} = \text{R} \times 2\pi, \text{ or } \frac{\text{D} \times \text{cot.}}{\text{cos.} \times 2\pi} = \text{R}.$$

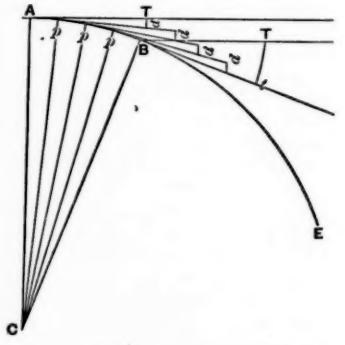
If D is divided by 86400 it will be the distance travelled in one second. Let this be denoted by V, which may be substituted in the equation for D, if 2π is also divided by 86400 thus,

$$\frac{\text{V} \times \text{cot.}}{\text{cos.} \times \frac{2\pi}{86400}} = \text{R}, \text{ or } \frac{\text{V} \times \text{cot.} \times 86400}{\text{cos.} \times 6.2832} = \text{R},$$

or $$13750.955\text{V.} \times \frac{\text{cot.}}{\text{cos.}} = \text{R}.$$

So that the tendency of a locomotive, running with the velocity of V on a straight line on the earth's surface at any point of latitude, is the same as that of one running at the same velocity upon a stationary curve whose radius is 13750.955V. $\times \frac{\text{cot.}}{\text{cos.}}$. To reduce this force to its equivalent in gravitation or pounds, it is only necessary to determine the centrifugal force on such a curve.

To establish a formula for the centrifugal force of a body moving in a circle let AT represent the tangent in which the body is moving when the resistance is applied at A to deflect it into the curve ABE, and let AB=AT be the distance in the curve moved in one second.



If the resistance were suddenly to cease at any point of the curve the deviation from the last tangent would continue with uniformity; but as the resistance is applied at every point *ppp* along the curve AB, and, the motion being uniform, at equal periods of time just as the impulse of gravitation is applied at equal periods of time along the path of a descending body, the deviation from the first tangent is constantly acceler-

ated. At each point an impulse is imparted at right angles with the last motion causing the increments of deviation *dddd*, whose sum at the end of the distance moved in a second is equal to Tt, the arc of the angle formed by the first and last tangent having BT, equal and parallel to AT, for radius. This arc is the deviation per second produced by the sum of the impulses during one second and which would remain uniform if they were to cease. If the force of gravitation would cause the same body to move towards the earth's center at a certain rate after it had been applied for this same length of time, with no other resistance than that of inertia, the impelling force, or weight, would be in the same proportion to this centrifugal force as the rate of falling is to the rate of departure from the first tangent at the end of the same period of time. As the velocity of a falling body at the end of the second is 32.22 feet per second the weight of the revolving body will be in the same proportion to the centrifugal force as 32.22 is to the arc Tt; which may be expressed by the equation $\frac{32.22C}{W}$ =Tt, in which C is the centrifugal force and W the weight of the moving body. The angle formed by the first and last tangent is equal to that formed by the two radii at the same points: therefore, the radius AC has the same proportion to the arc AB as the tangent AT=BT has to the arc Tt. And since the arc AB is equal to the tangent Bt we have the proportion AC : AB :: Bt=AB : Tt or AC×Tt=AB^2. AB being the distance moved in one second is the velocity, to be denoted by V: and AC being the radius of the circle is denoted by R. Then R ×Tt=V^2 or Tt=$\frac{V^2}{R}$. In the equation $\frac{32.22C}{W}$ =Tt, substitute for Tt its value $\frac{V^2}{R}$; then $\frac{32.22c}{W}=\frac{V^2}{R}$ or C=$\frac{V^2\times W}{32.22R}$. Now let the radius which has been determined as that of a stationary curve causing the same lateral resistance as that of the straight line on the rotating earth be substituted for R in this equation. It will then be

$$c=\frac{W\times V^2}{32.22\times 13750.955V\times\frac{\text{cot.}}{\text{cos.}}}$$

$$\text{or } \frac{W\times V^2}{443055.75}\times\frac{\text{cos.}}{\text{cot.}}=C.$$

which is the force with which a locomotive tends to leave the straight track at any degree of latitude.

Let V, the velocity of the locomotive, be assumed at 60 feet per second=41 miles per hour, and W, the weight at 80,000 lbs.

In latitude 20° $\frac{80000}{60\times 356.605}$ = 3.74 lbs.= lateral impulse.

In latitude 40° $\frac{80000}{60\times 190.340}$ = 7.00 lbs. = lateral impulse.

In latitude 60° $\frac{80000}{60\times 141.165}$ = 8.04 lbs. = lateral impulse.

In latitude 90° $\frac{80000}{60\times 123.070}$ =10.83 lbs.= lateral impulse.

As every part of a line deflecting about a fixed point is itself deflecting about any of its own moving points at the same rate and in the same direction, the ends towards the fixed point deflecting in a contrary direction from the motion while the opposite ends deflect in the same direction, it follows that all the deflections north of the equator must be towards the left and all those south of the equator towards the right; which would cause the locomotive to tend to the right in northern latitudes and to the left in southern latitudes.

So far the track has been supposed to be located upon a north and south line, according to the pretended observations. But it is apparent that the effect is the same upon any horizontal line at the same latitude, for all such lines that can be drawn upon the surface of the sphere radiating from a common point must all deflect at the same rate like the spokes of a wheel, confined to the same horizontal plane. So that a locomotive upon a track located east and west will be subject to the same lateral impulse as that on the one north and south, the only difference being that of quantity according to latitude and direction according to position with regard

to the equator. The motion that the lateral impulse was confined to a north and south track, arises from the fact that it is easy to perceive a difference in rate of motion on two circles of different latitude, and that a body endowed with the velocity of one being transferred to the other would encounter the resistance arising from the difference of velocities. But this variation of the circles of the same conical zone corresponds exactly with variation in lateral velocity of equally distant points on the deflecting tangent which describes the cone, or any other lines belonging to the same small horizontal material plane which follows the describing tangent, and which rotates to the same degree which the tangent deflects. The rotation of the small horizontal planes which compose the surface of the sphere is the sole cause of the lateral impulse to moving bodies upon it, and the difference of the conical circles is the measure of the deflection of all these lines alike. Therefore as the observations of derailments have been credited solely to north and south tracks, this proves them to be purely imaginary. And as in every case of derailments the cause must be one involving a vastly greater force than this lateral impulse, it could have no appreciable effect in determining the direction.

SOME PRACTICAL NOTES ON THE SEASONING OF BUILDING WOODS.

From "The Builder."

THERE have, perhaps, been as many papers written relative to the seasoning of wood as have been contributed to the discussion of any other purely technical question.

The subject is one to which theorists in particular have devoted a large share of their attention. The matter, indeed, has, for the greater part, been theoretically rather than practically discussed, and whilst an extreme degree of attention has been directed to issues of comparative unimportance, others of material importance have been overlooked. It is hoped by this paper to supply some of the deficiencies which exist.

Commencing upon our inquiries, it is perhaps as well, in the first instance, to inquire what the seasoning of wood really amounts to. It appears to be a prevalent notion that some extraordinary chemical effect upon the juices and saps of wood is brought about by the process of seasoning. Such theories as are, however, held by some scientists possess very little interest to those who make use of wood only as a constructive material.

For practical purposes a given effect is required. Wood is required to be seasoned, and what is mainly wanted to be known upon the subject is, what is the best and most economical method of obtaining the desired result? By what is commonly known as seasoning wood is meant nothing more than drying wood.

Wood is to be dried only by the moisture being evaporated from it. It is an elastic material, and in degrees as the moisture exudes so it will shrink. It is also by nature somewhat absorbent, and it will therefore, under certain conditions, attract and retain moisture. For instance, if highly dried wood-work be fixed in a thoroughly damp house, the wood will absorb some moisture, and being of an elastic nature, it will re-expand just in proportion to the amount of moisture it has absorbed. When the house has become dry, the wood will also have dried, and the result of the expansion and afterwards of the contraction will be a number of open joints. For these open joints the joiner may receive much censure, although it is highly possible that he may not have been at all in fault. Indeed, if the wood he has used has been highly dried it will, in all probability, have absorbed more moisture than would have been the case if it had only been partially dried, and so when it shrinks again the openings left will in such a case be wider.

Woodwork to be fixed in a newly-

erected and, consequently, damp building, should have received a coating of paint before being fixed, as, in such a case, it will not absorb moisture.

These remarks may suggest the inquiry why unpainted woodwork does not continually undergo a process of expansion and contraction. The question is not a difficult one to reply to, although it has not usually been answered in our way.

It is well enough known that wood, in a living state, forms a skin over any exposed portion. Thus, if a branch be lopped off a tree, the injured limb for a time will bleed its sap. Nature, however, supplies its own protection against injury, and a thin skin is soon formed over the injured part, which stays the bleeding. It is the same, to some extent, with dead as with living wood. Flooring-boards, for example, which have been laid down for ten or almost any other number of years, will, if planed over, enter upon a second process of shrinking or expansion, according as they are situated in a dry or humid atmosphere. To season wood, then, is simply to dry it, and that means nothing more than evaporating the moisture from it. For all practical purposes, therefore, we have only to inquire,

First, what amount of seasoning is required?

Secondly, what is the best way to accomplish the seasoning process?

The issues are, consequently, narrowed very considerably.

When we ask ourselves what amount of seasoning is necessary, we must hold steadily in view the requirements of the different purposes for which wood is used.

And, first of all, it should be clearly explained that foreign-sawn building woods undergo a limited amount of seasoning before they leave the other side of the water. It would not at all do to store green wood into the unventilated holds of ships, for if such a thing were done, the wood would at once commence to "sweat." In a short time it would become covered with a mildew, and the sap-wood would become permanently discolored; it would then have depreciated very much in commercial value. To avoid such an occurrence the wood is for a time seasoned abroad under large wooden sheds before being shipped. It is in consequence of being seasoned under sheds that it usually reaches us of such an excellent color and clean appearance.

The bulk of the foreign building wood imported is used for bearing or joisting purposes. For such purposes it is not necessary that it should be at all seasoned beyond the preliminary seasoning it undergoes previously to shipment. It is a common error to suppose that old wood is in any way better for bearing purposes. A kind of vague idea has generally existed that old wood is necessarily seasoned, and that it is better that it should be seasoned.

In a vague way this view is held to, and yet no one can say in what way *bearing wood* is benefited by being seasoned. As a matter of fact, it is usually more or less *injured* by being old.

At the timber-yards, deals are, as a rule, most indifferently piled. They are not generally in any way protected from the weather, nor are they stored in such a manner as will insure any water which falls upon them running quickly off again. If not piled so that air can circulate through them, and if moisture be present, decay will rapidly supervene.

If the sun's rays be allowed to approach the ends and the sides of deals, a splitting process occurs. Should wet weather then follow, these cracks will become filled with moisture. When frost follows upon wet the splitting process continues, and the deals often become in consequence much shaken. Old deals will have endured these contingencies, and it may be taken for granted that they will not have escaped scatheless. Consequently it is better on the whole to employ perfectly new wood for all kinds of bearing work.

Wood, however, that is intended to be made up into joinery work must be seasoned. There are two matters to be considered in relation thereto. First, the wood has to be dried. Secondly, it is not to be injured in the process of drying. The injuries that can be done to the material are those to be caused by the sun, rain and frost, and by the dirt if it be stored in the immediate neighborhood of a large town. The only sufficient protection against these influences is a suitable shed for the wood to be placed under. The sides and the ends of the shed must, of course, be open to admit of the wind circulating freely

through the deals or boards required to be dried. Thus all that is required is a suitable roof. Such a roof should be exceedingly flat, so as to admit of the convenient piling away of the deals or boards under it. It should be made to well overhang so as to serve in the double capacity of a sunshade and an umbrella to keep off the sun and the rain. It should also be made capable of being raised or lowered at will, so that it can be placed just over any pile. It should be built in the most exposed part of the builder's premises, for no factor is so important in relation to the seasoning of wood as is the wind. If it blows a gale, so long as it does not upset the piles, it is all the better for the purpose of seasoning.

Deals and boards, when placed for seasoning, should alike be separated at regular intervals by evenly sawn strips of wood. It is better that boards should be seasoned when lying in a horizontal position, because they are then less likely to twist or warp whilst being dried. The long and the short pieces should be piled separately, and in no case should a long deal or board be allowed to overhang without any support, or it will be certain to twist. It is worth while taking care in this respect of those intended to be used for joinery purposes, that are required to be seasoned at all.

Flooring-boards will probably have to be seasoned in the open, because room will not, probably, be found for them under the drying-sheds. If such a thing is possible, then it is all the better, but it is a matter of less importance that flooring-boards should be protected from the weather than any other kind of wood that is required to be dried. When flooring-boards are to be dried in the open, it is better that they should be "perched" on their ends rather than laid out in a horizonal or triangular form. When piled in a triangular form the boards are exceedingly liable to sway at their centers, and thus they will be likely to dry in a twisted form. They also hold at times a considerable amount of water in such a case. It is a good plan, when stacking flooring-boards on a perch to pile the boards in pairs with their face sides together, as in such case the faces of the boards are kept clean, and thus the necessity of replaning the faces of the boards when laid is rendered unnecessary. Seasoned boards will enter upon second shrinking or expansion process if their newly-formed skins be removed with a planing-iron. There are those persons who are in the habit of drying their flooring-boards before passing them through the planing-machines. This is done so as to have a perfectly clean board when laid, but, for the reasons given, the result is not satisfactory, and further than this, clean faces can be preserved by adopting the practice of stacking the boards in pairs to dry.

For certain processes, such as mould-making, for instance, wood must be highly dried, and in such cases "stoving" may be resorted to with advantage. The main reform, however, required to be effected in respect to the seasoning of building wood appears to be the more general erection of suitable and sufficient drying-sheds, having open sides and flat, movable roofs. Doubtless the erection of such sheds necessitates the expenditure of a considerable amount of money, but it is as well to bear in mind the admitted adage, that what is worth doing at all is worthy of being done well. There can be no doubt but that most consumers of wood suffer heavily through losses incurred in consequence of the depreciation of stock which has not been sufficiently well cared for. A well-designed and suitably situated drying-shed will permit of the wood which is stored under it being perfectly seasoned, and yet kept free from grit and dust, whilst the havoc which the sun, the rain, and the frost usually commit upon wood that is undergoing the process of being dried will be altogether avoided.

It should be well borne in mind that wood often suffers very much in consequence of its having been stored away in the sale timber-yards without any sort of protection, and it is to be hoped that ere long some protective provisions, in the shape of sheds, will be brought into general requisition by the timber merchants. Until such protective provisions do become generally established, we would strenuously advise builders to buy, when possible, newly-arrived stock, and to take care themselves of that portion which they intend to devote to joinery or other similar purposes.

THE KINETIC THEORY OF GASES.

By HENRY T. EDDY, C.E., PhD., University of Cincinnati.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

1. INTRODUCTORY.—The atomic theory of the constitution of matter in general is an hypothesis which originated so far as we know among the Greek philosophers* but was held by them merely as part of certain philosophical and metaphysical systems. The development of the atomic and molecular theory by comparison with the facts of nature is a part of modern science with which chemistry has been largely concerned. But the Kinetic theory of the constitution of gases is a branch of this theory which does not lie in the domain of chemistry but almost exclusively in that of mathematical physics. It is a working hypothesis as to the physical nature of gases, and attempts to conceive of and account for their properties by a definite mechanical arrangement of atoms and molecules.

It does not apply to solids and liquids since they are regarded as possessing a more intricate arrangement mechanically considered, and, indeed, gases can have the supposed simplicity of construction only when not too dense.

It may be that the hypothesis is not a presentation of facts at all true to nature, it may be that the ideas embodied in the supposed atomic and molecular constitution of matter are in fact untrue; nevertheless the multitudinous cases of agreement between the mathematical conclusions drawn from the theory with the facts of nature leave hardly the shadow of a doubt that the mathematical relations arrived at by this theory expressed substantially actual laws of nature. The reason of this may be that the mathematical expressions are of more general nature than the special physical hypothesis which they have been used to express, and thus are possibly true while the hypothesis is untrue.

This theory is not merely an endeavor to get rid of metaphysical and other difficulties respecting the constitution of gases by predicating phenomena of molecules and atoms because they are so small that it may be quite impossible to refute assertions made respecting them. On the contrary, something of the kind assumed by this theory has place in nature as is evident from spectroscopic and other evidence. At all events, the theory is in many points so nearly coincident with experimental phenomena that it is deservedly regarded as a part of modern science, occupying a place analogous to that of the undulatory theory of light, as a useful, perhaps indispensable instrument of investigation. The deviations of theory from experiment are in many cases fully explained as in other applications of mechanics to complicated material systems, by the impossibility of solving the problem in its most general and complicated form, for which reason we are obliged to content ourselves with a more simple result, which is only a close approximation to the case which we had wished to treat.

The first step of importance in the mathematical investigation of this theory was taken by Daniel Bernoulli, who showed in 1738 how the pressure exerted by gases may be accounted for on this theory.

But it is the investigations of Clausius from 1857 onward, followed by the brilliant discoveries of Maxwell in 1860, and more recently the extended researches of Boltzemann, published during the last ten or fifteen years in the *Wiener Sitzungsberichte*, which have principally awakened the attention of physicists to the importance of the theory, and led to its development and experimental confirmation. Others have made important contributions to the theory, among whom may be mentioned Krönig, whose first paper, in 1856, anticipated some of Clausius results; Joule, whose first paper appeared in 1851; Nilyer and Watson whose valuable treatises* have done much

* See article entitled "Atom," in the Encyclopedia Britannica, Vol. III, 9th edition.

* Die Kinetische Theorie der Gase, pp. 338. D. O. E. Meyer, Breslau, 1877. The Kinetic Theory of Gases, pp. 51. H. W. Watson, M.A., Oxford, 1876.

to render the theory accessible to the student.

2. General Statement of the Theory.—The Kinetic theory assumes that any given volume of any of the gases consists of a great number of separate particles which are almost or entirely independent of each other. These particles are usually identified with chemical molecules. In a state of equilibrium these molecules are supposed to be in rapid and independent motion in straight lines. The unoccupied space between the molecules is assumed to be very large compared with that actually occupied by the molecules themselves. This might be illustrated by comparing the molecules to a swarm of insects on the wing.

It must happen that frequent encounters will occur between molecules moving in this manner.

The length of the path between the successive encounters of the same molecule with any others is called its free path. The length of the free path is, therefore, very great compared with the dimensions of the molecule itself, and the time occupied by an encounter is very small compared with that consumed in describing a free path.

The theory further supposes that the sensible heat of a gas exists in it in that form of energy called *vis viva* and is the total energy of progressive motion of the molecules, so that when the temperature of a gas is augmented it is assumed that the average velocity of its molecules is so increased that their augmented energy of translation corresponds to the increase of temperature.

As to the velocity of the molecules, Clausius' first investigations were made on the supposition that in a simple gas, *i.e.*, one which is not a mixture, all the molecules have the same velocity, and the results obtained on this supposition are for many purposes correct, the velocity supposed being simply such a mean velocity as would make the progressive energy equal to the sensible heat.

But Maxwell was the first to point out that whatever be the original distribution of velocities there is but one final distribution which can be permanent, a distribution whose only law is the law of chance, but one which is susceptible of mathematical statement and one in the average of a very large number of molecules must in all cases give the same result.

The whole path of a molecule will then consist of a broken, zigzag line, the straight portions of which will be of unequal length and in every possible direction, the length, direction, and velocity being dependent upon the laws of chance. Hence, after the lapse of even a considerable interval of time, a molecule will in general be found at an inconsiderable distance from its initial position.

It has often been furthermore assumed that the molecules are perfectly elastic or perfectly hard, or both, and objections have been made to the theory on this score, but assumptions of this nature seem not to be a necessary part of the theory.

3. Clausius' Theorem Respecting the Virial of Stationary Motion.—A material system is said to be in stationary motion when its constituents do not continually depart from their initial position, and their velocities do not continually recede from their initial values. The molecules of a gas in equilibrium constitute such a system. For the sake of clearness let the system of molecules under consideration to those constituting a one unit of mass of gas contained within impervious, immovable walls, although the same laws will evidently hold in case the walls are merely imaginary boundaries conceived as separating the unit of gas under consideration from other surrounding units.

Let m be the mass of any molecule of this gas and xyz its rectangular co-ordinates referred to any origin; let XYZ be the components along the axes of xyz respectively of the resultant of all the external forces acting upon the molecule m, taken as positive when they tend to increase xyz respectively.

Now, by the principles of the differential calculus, we have the identical equation

$$\frac{1}{2}\frac{d^2}{dt^2}(x^2)=\frac{d}{dt}\left(x\frac{dx}{dt}\right)=\left(\frac{dx}{dt}\right)^2+x\frac{d^2x}{dt^2}\ . \quad (1).$$

as may be seen by performing the differentiation expressed in the first two terms. But by the fundamental principles of dynamics we also have

$$m\frac{d^2x}{dt^2}=X \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Substitute from (2) in (1) and transpose,

$$\therefore \quad \frac{m}{2}\left(\frac{dx}{dt}\right)^2=\frac{m}{4}\frac{d^2}{dt^2}(x^2)-\frac{x}{2}X \quad . \quad . \quad (3).$$

The first member of (3), being one-half of the product of the mass by the square of the velocity, expresses the energy of progressive motion of m parallel to x at any instant when its co-ordinate is x and the moving force along x is X.

To find the average value of this energy during any interval t, (3) must be multiplied by dt, integrated between the limits 0 and t, and then the result divided by t, this being the ordinary process for finding the average or mean value of any function for the time t.

$$\therefore \quad \frac{m}{2t}\int_0^t\left(\frac{dx}{dt}\right)^2dt=\frac{m}{4t}\left\{\left(\frac{d}{dt}(x)^2\right)-\left(\frac{d}{dt}(x^2)\right)_0\right\}-\frac{1}{2t}\int_0^t x\times dt \quad . \quad (4).$$

The first and last terms of (4), as just stated, are expressions for the average or mean values of the corresponding terms in (3). But the quantity within the square brackets is of a different nature; it is the difference between the final and initial values (as expressed by the subscripts) of a function whose final and initial values may, if t be properly chosen, be equal. But it is unnecessary so to choose t. For, by reason of the divisor t, it appears that if t be taken sufficiently large, the term under consideration vanishes even though the final and initial values are not equal, since they cannot recede indefinitely from each other. That they cannot so recede appears from the identical equation, obtained as was (1)

$$\frac{d}{dt}(x^2)=2x\frac{dx}{dt} \quad . \quad . \quad . \quad . \quad . \quad (5).$$

From which it appears that the term is dependent upon the products of quantities (co-ordinates and velocities) which by the definition of stationary motion could not recede indefinitely from their initial values.

Let now xyz denote no longer the actual co-ordinates of m, but instead the average values of these quantities during the time t, and similarly for XYZ, and let $x'y'z'$ be the corresponding velocities along the axes. Hence, we may write (4), and the two similar equations with respect to the axes of y and z, as follows:

$$\left.\begin{aligned} mx'^2&=xX\\ my'^2&=yY\\ mz'^2&=zZ\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6).$$

in which, as just stated, the variables express average values during the interval t, which may be taken so large as to give sensibly constant values.

Let the unit of gas under consideration consist of n molecules, which may have equal or unequal masses.

Like equations apply to all the members of the system of molecules which may be distinguished as m_1, m_2 . . . m_n. Suppose these equations formed and take their sum.

$$\therefore \quad \tfrac{1}{2}\Sigma_1^n m(x'^2+y'^2+z'^2)= -\tfrac{1}{2}\Sigma_1^n(xX+yY+zZ) \quad . \quad . \quad (7).$$

The first member of (7) is the total Kinetic energy of the system, and the last member is called the virial of the system, and depends upon the average forces acting upon the particles and their position.

The theorem which has been now demonstrated may be thus stated: The mean kinetic energy of a system in stationary motion is equal to its virial.

4. External Virial and Gaseous Pressure.—In case the molecules neither attract nor repel each other, and no forces act upon them except the external pressure exerted by the walls of the vessel containing them, the virial of this pressure may be computed as follows: Let the closed surface containing the unit of mass of the gas under consideration be of any shape whatever. It is evident that the pressure of the enclosed gas in equilibrium will be normal to the surface. Let p be the intensity of this pressure per unit of area, and let dS be the element of area of the enclosing surface, and l the cosine of the angle which the normal to it makes with the axis of x.

$$\therefore \quad ld\mathrm{S}=dydz \quad . \quad . \quad . \quad . \quad (8).$$

$$\therefore \quad -\frac{1}{2}\Sigma_1^n xX=\frac{1}{2}\int xpld\mathrm{S} =\frac{p}{2}\int\int xdydz \quad . \quad . \quad (9).$$

in which the integration in the second member is to be extended over the enclosing surface alone, for only those molecules are acted upon by the pressure which are at the surface. The last integral in (9) expresses the whole volume v enclosed within the surface and does not depend upon the kind of molecules under consideration. The sign of p is opposite to that of X, because p causes the molecules to approach each other, while X tends to make them recede from each other.

$$\therefore \text{ by (9) } -\tfrac{1}{2}\Sigma_1^n x\mathrm{X}=\tfrac{1}{2}pv \quad . \quad . \quad (10).$$

with similar equations for y and z, which being added to (10) give the total external virial due to the pressure.

$$\therefore \text{ by (7)} \tfrac{1}{2}\Sigma_1^n m(x'^2+y'^2+z'^2)=\tfrac{3}{2}pv. \quad (11).$$

in which v is the volume occupied by the unit of gas considered, and n is the number of molecules in it, and (10) expresses the relation existing between the total energy of the progressive motion of the molecules to the volume and pressure in case the molecules neither attract nor repel each other.

Now, suppose that the unit of gas under consideration consist of molecules which are all of the same kind, *i. e.*, it is not a mixture of two or more gases, then $m_1=m_2$, etc. $=m_n$, $x'_1{}^2=x'_2{}^2=x'^2{}_n$, etc., etc., since each molecule will then have the same mean component velocities. Let $v^2=x'^2+y'^2+z'^2$, then (11) may be written in the form

$$\tfrac{1}{2}num^2=\tfrac{3}{2}pv \quad . \quad . \quad . \quad (12).$$

in which v^2 is the mean square of the velocity of the molecules, and the first member is the mean energy of translation of all the molecules in the unit of mass of the gas. If q is the mass of a cubic unit of this gas, then

$$nm=qv \quad . \quad . \quad . \quad . \quad (13).$$

in which either side of the equation expresses the unit of mass of the gas under consideration. Eliminate nm from (12) and (13).

$$\therefore \quad qv^2=3p \quad . \quad . \quad . \quad (14).$$

an equation which enables us from the known values of p and q to compute the value of v.

Let us consider the atmospheric air for the moment to be sufficiently near by a homogeneous gas for our purpose, though in fact it is a mixture whose different molecules differ slightly in mass.

Take the mean atmospheric pressure at $p=2116.4$ lbs. per square foot, and the product of the weight of 1 cubic foot of air at 32° F., at this pressure (=0.080728 lbs.) by the acceleration of gravitation, as the mass $q = 0.80728\times 32\frac{1}{6}$ from which we obtain

$$v=1591 \text{ ft. per sec. for air,}$$

a velocity which does not from that of a rifle ball, and is about one and a half times that of sound.

The corresponding velocities for the simple gases can be found in like manner. Besides the above value for air, Clausius gave, in 1857, the following values:

For oxygen, $v=1512$. ft.
For nitrogen, $v=1614$. ft.
For hydrogen, $v=6048$. ft.

5. Internal Virial of Molecular Attraction and Repulsion.—The magnitude and algebraic form of the internal virial affords pretty decisive evidence as to the existence or non existence of molecular attractions or repulsions of appreciable amount. In order to compute it let r be the distance between any two molecules m_1 and m_2, and let $f(r)$ be the function of r, which expresses the law of the force, the value of which is positive in case it be a repulsion, and negative for an attraction.

$$\therefore \quad x_1\mathrm{X}_1+x_2\mathrm{X}_2=x_1\frac{x_1-x_2}{r}f(r)$$
$$+x_2\frac{x_2-x_1}{r}f(r)=\frac{(x_2-x_1)^2}{r}f'(r) \; . \; (15).$$

with two similar equations for the remaining axes of y and z for each pair of molecules, in which $\frac{x_1-x_2}{r}$ is the cosine of the angle which the repulsion which m_2 exerts on m_1 makes with the axis of x, etc., etc.

If we take the summation of the three equations for each pair of molecules in the system by taking with each of the n molecules all the others, it is evident that a pair of terms containing the quantities $(x_2-x_1)^2$ and $(x_1-x_2)^2$ will appear and other pairs of the same form, so that the summation taken in this manner will be twice the result sought.

Furthermore, $(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2=r^2$, etc.

$$\therefore \quad -\tfrac{1}{2}\Sigma_1^n(xX+yY+zZ)=-\tfrac{1}{4}\Sigma_1^n\Sigma_1^n rf(r) \quad . \quad . \quad . \quad (16).$$

in the last member of which the value of $\Sigma_1^n rf(r)$ is to be found for each of the n molecules by taking with each one all the others, and then these separate summations are to be added to form the sum total. The last member of (16) is often written with the coefficient $\frac{1}{2}$, in which case $\Sigma rf(r)$ is to be taken for all possible different pairs of molecules.

It is evident that the expression for the internal virial in (16), as also that for the external in (11) depends in no way upon the axes chosen for making the summation, for the results obtained are independent of the axes. But in general the value of the internal virial in (16) depends upon the form of the surface enclosing the gas under consideration, unless a suitable special form be assigned to the friction $f(r)$ which expresses the law of the force. Now within the limits of ordinary experiment the form of the vessel exerts no appreciable influence upon the behavior of the gas, for let the values of the external and internal virial from (11) and (16) be substituted in (7).

$$\therefore \quad \tfrac{1}{2}\Sigma_1^n m(x'^2+y'^2+z'^2)=\tfrac{3}{2}pv \quad -\tfrac{1}{4}\Sigma_1^n\Sigma_1^n rf(r) \quad . \quad . \quad . \quad (17).$$

First suppose the unit of mass of the gas enclosed within a sphere of volume v, and next within a long narrow tube of the same volume. The first member, which is the total energy of translation of the molecules is not changed if the temperature is not; hence unless the last term be constant p must vary. But p being invariable within the limits of ordinary experiment the last term is constant.

$$\therefore \quad rf(r)=a, \quad \therefore \quad f(r)=\frac{a}{r} \quad . \quad . \quad . \quad (18).$$

in which a is some constant. But the law of force expressed by (18), which is a repulsion inversely as the distance would cause the pressure to be greater in the more distant parts of the vessel. Hence within the limits of experiment $a=0$ and the internal virial vanishes.

In the computation of the internal virial the virial due to encounters between the molecules has been neglected, for on account of the assumed minute size of the molecules its amount may be seen to be small as follows: In an encounter between m_1 and m_2, $X_2=-X_1$

$$\therefore \quad x_1X_1+x_2X_2=X_2(x_2-x_1) \quad . \quad . \quad . \quad (19).$$

in which (x_2-x_1) is the projection upon x of the distance between the centers of the molecules at the instant of encounter, which has been assumed to be small, an assumption which will be justified later. We shall also endeavor hereafter to take into further consideration the effect of very feeble forces acting between the molecules as well as that of the encounters.

Several demonstrations have been given of (11) besides that by means of the virial, all of them assuming, of course, that no molecular attractions or repulsion exist. The special advantage of this demonstration is that certain obscurities are avoided, and no assumption is made as to the elasticity or want of elasticity of the molecules; and further, in (17) is a term dependent upon the internal forces which can be employed when it shall seem best to consider them.

6.—Extension of Clausius' Theorem of the Virial to Rotations.—The author has ventured to propose the following investigation of the virial of rotations and rotary velocities of bodies in stationary motion analogous to that given by Clausius for translations.

Let a, b, c, be the moments of inertia of any molecule m with respect to its principal axes. Let

$$\frac{d\theta}{dt}\cdot\frac{d\varphi}{dt}\cdot\frac{d\psi}{dt}$$

be the angular velocities about these principal axes, then are θ, Φ, ψ angular co ordinates with reference to same initial lines yet to be assumed.

Let L, M, N be respectively the components about these axes of the resultant of all the couples acting upon the molecule.

Then by Euler's equations of rotary motion

$$a\frac{d^2\theta}{dt^2}=(b-c)\frac{d\varphi}{dt}\cdot\frac{d\varphi}{dt}+L \quad . \quad . \quad . \quad (20).$$

We also have the identical equation,

$$\frac{1}{2}\frac{d^2}{dt^2}(\theta^2)=\frac{d}{dt}\left(\theta\frac{d\theta}{dt}\right)=\left(\frac{d\theta}{dt}\right)^2+\theta\frac{d^2\theta}{d^2t}. \quad (21).$$

as appears by performing the differentiation expressek in the first two terms.

Substitute from (20) in (21) and transpose

$$\therefore \quad \frac{a}{2}\left(\frac{d\theta}{dt}\right)^2=\frac{1}{4}\frac{d^2}{dt^2}(\theta^2)-\frac{1}{2}(b-c)\theta\frac{d\varphi}{dt}\frac{d\psi}{dt}-\frac{1}{2}\theta\mathrm{L} \quad . \quad . \quad . \quad . \quad (22).$$

The first member of (22) being one-half of the product of the moment of inertia about one of the principal axes by the square of the angular velocity about that axis expresses the energy of rotation about that axis.

Now find the average value of this energy during the interval t as was done in (3), and let θ, θ', L, &c., be now used to express mean values of θ. $\frac{d\theta}{dt}$.L, &c., for the molecule m.

We then obtain from (22) an equation which with the two similar ones with respect to the remaining principal axes may be written

$$\left.\begin{aligned} a\theta'^2&=-\theta\mathrm{L}-(b-c)\theta\varphi'\psi' \\ b\varphi'^2&=-\varphi\mathrm{M}-(c-a)\varphi\psi'\theta' \\ c\psi'^2&=-\psi\mathrm{N}-(a-b)\psi\theta'\varphi' \end{aligned}\right\} \quad . \quad . \quad (23).$$

in which it is to be noticed that the quantities θ'^2, φ'^2, ψ'^2, in the first members are not the squares of the quantities represented by θ', φ', ψ' in the second members, for in the first members the quantities are the mean squares of all the numerical values of the angular velocities and no account is, therefore, taken of their signs. But the mean values of θ', φ', ψ' in the second members of (23) are each zero, because on the average there must apparently be as many and as large positive as negative angular velocities about any one of the axes. The last terms of (23), therefore, vanish, whatever may be the mean values of θ, φ, ψ.

Now let

$$\mathrm{L}=\rho_1\mathrm{X}, \quad \mathrm{M}=\rho_2\mathrm{Y}, \quad \mathrm{N}=\rho_3\mathrm{Z} \quad . \quad . \quad (24).$$

in which Y, X, Z have same significations as in (6) and (7), there is ρ_1 the mean value of the arm of the couple L, etc.

Also let

$$\theta\rho_1=x, \quad \varphi\rho_2=y, \quad \psi\rho_3=z \quad . \quad . \quad . \quad (25).$$

which equations will fix the initial lines from which θ, φ, ψ are measured.

Hence by (24) and (25) we may write (23) thus—

$$\left.\begin{aligned} a\theta'^2&=-\theta\mathrm{L}=-x\mathrm{X} \\ b\varphi'^2&=-\varphi\mathrm{M}=-y\mathrm{Y} \\ c\psi'^2&=-\psi\mathrm{N}=-z\mathrm{Z} \end{aligned}\right\} \quad . \quad . \quad (26).$$

Now take the sum of equations (26) for n molecules,

$$\therefore \quad \tfrac{1}{2}\Sigma_1^n(a\theta'^2+b\varphi'^2+c\psi'^2)=-\tfrac{1}{2}\Sigma_1^n(x\mathrm{X}+y\mathrm{Y}+z\mathrm{Z}). \quad . \quad (27).$$

the first member of which is the total energy of rotation of the molecules of the gas, and the second member is the virial which is to be computed as in (7).

$$\therefore \text{ by (10)} \quad \tfrac{1}{2}\Sigma_1^n a\theta'^2=\tfrac{1}{2}pv \quad . \quad . \quad (28).$$

$$\therefore \quad \tfrac{1}{2}\Sigma_1^n(a\theta'^2+b\varphi'^2+c\psi'^2)=\tfrac{3}{2}pv \quad . \quad (29).$$

From (11) and (29) it appears that in general the mean energy of translation is equal to the mean energy of rotation, and that the total kinetic energy of a gas is one-half due to progressive motion and one-half to rotation.

We have said that this is true in general, we must, however, except the cases in which one or more of the couples L, M, N vanish identically L, M, N may evidently all three so vanish when the molecules are in effect smooth spheres, and a single one of them may vanish when the molecules are in effect smooth solids of revolution.

In case $\mathrm{L}=\mathrm{M}=\mathrm{N}=0$ we must have in (24) $\rho_1=\rho_2=\rho_3=0$, and Eqs. (25) are then impossible, but by recurring to (23) we see that the energy of rotation about each axis vanishes.

In case only $\mathrm{L}=0$, the total energy of rotation is

$$\tfrac{1}{2}\Sigma_1^n(b'\varphi+c\psi'^2)=pv \quad . \quad . \quad . \quad (30).$$

From (11) and (30) it appears that in this case the energy of rotation is two-thirds that of translation, and the total kinetic energy is two-fifths of it rotary and three-fifths progressive.

It does not seem necessary to suppose that the preceding investigation applies only to molecules in which abc remain constant, although it appears not unreasonable hypothesis to suppose that the molecules of any substance are unchanged by change of state of aggregation, and that whether a substance be solid, liquid, or gaseous, the molecules of which it is

composed may be regarded as nearly rigid, *i. e.*, incapable of deformation to any considerable extent by finite forces, which is equivalent to supposing abc constant.

It should be further noticed that in case $b=c$ it is not necessary to suppose as we have done that the mean values of θ', φ', ψ' vanish, for we shall evidently then have $\varphi'=\psi'$; and the two equations $b=c$, and $\varphi'=\psi'$ are sufficient to enable us to derive (26) and (27) from (23). The hypothesis that $b=c$ must in many cases be correct, and it may perhaps be shown in all cases not to be far from true, when (27) will be approximately true from this reason alone.

7. Temperature, Volume and Pressure. —The first member of (11) is the mean energy of the progressive molecular motion of a unit of mass of gas: and since heat is known to be energy, this energy is ordinarily assumed to be the quantity of sensible heat in the unit of gas, as stated in Art. 2.

It would, however, be apparently more correct to assume that the sensible heat is the total kinetic energy, rotary and progressive. But since it has been shown that in any given gas these quantities have a fixed ratio to each other, this assumption makes no change in the equations employed. The mean progressive energy in either supposition is proportional to and measured by the absolute temperature τ of the gas, *i. e.*, τ is to be reckoned from a state devoid of energy.

Let k be the specific heat of the gas at constant volume, measured in ft. lbs., then the statement just made is expressed by the equation

$$\tfrac{1}{2}\Sigma_1^n m(x'^2+y'^2+z'^2)=ak\tau \quad . \quad . \quad (31).$$

in which a denotes what fraction of the total heat $k\tau$ contained in the gas exists in it in the form of progressive molecular motion.

If, in (17), we let $f'(r)=-\mathrm{R}$, then $+\mathrm{R}$ expresses the mean attractive intermolecular force.

If also we let a single sign of summation denote the sum with respect to all possible pairs of molecules, we obtain on substituting (17) thus modified in (31),

$$ak\tau=\tfrac{2}{3}pv+\tfrac{1}{3}\Sigma r\mathrm{R} \quad . \quad . \quad . \quad . \quad (32).$$

in which it should be noticed that for a given value of pv the greater the attraction between the molecules the higher the temperature, so that a perfect gas, *i. e.*, one having no intermolecular attractions, is at a higher temperature than one whose molecules attract; a remarkable result, the bearing of which seems heretofore to have been overlooked, but which will appear later.

If the effect the mutual attractions and repulsions be neglected, we may write (32) in the form

$$pv=c\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad (33).$$

where for brevity $c=\frac{2}{3}ak$.

It is seen that (33), which is here derived from theoretical considerations alone, is the equation expressing the empirical law of Gay Lussac for the so-called perfect gases, and it also includes Boyle's law. We may therefore consider (33) to express the laws obeyed by a system of molecules having no mutual attractions or repulsions, and moving in a space infinitely larger than that occupied by the molecules themselves; if, however, an appreciable fraction of the whole space in which they move is occupied by the molecules themselves, the effect of encounters between them will, on the whole, be that of a feeble repulsion; for owing to their assumed hardness their centers cannot approach within a certain distance of each other. All the minute interactions between the molecules, attractive or repulsive, which interfere with the rigorous exactness of (33) are included in the last term of (32) which seems to be of such importance to the exactness of the theory as to demand more close scrutiny than it has heretofore received.

Let the temperature of the gas be augmented by the small amount $d\tau$, the remaining variables in (32) will also suffer variations expressed by the equation

$$ak'd\tau=\tfrac{2}{3}d(pv)+\tfrac{1}{3}\Sigma d(r\mathrm{R}) \quad . \quad . \quad (34).$$

in which, considering for the moment the interactions of a single pair of molecules only,

$$d(r\mathrm{R})=\mathrm{R}dr+rd\mathrm{R} \quad . \quad . \quad . \quad . \quad (35).$$

R and r expressing mean values for the pair considered.

The term $\mathrm{R}dr$ in (35) is evidently the work performed in increasing the distance of the pair of molecules by the amount dr against their mutual attraction

R, and is an amount of work which will then exist in the gas in the form of potential energy.

The term $rd\mathrm{R}$ in (35) expresses the amount of variation of energy involved in the variation of the mean attraction. In case R is a function r, $d\mathrm{R}$ will depend upon r and dr. But R may be also a function of τ, as seems quite evident from the phenomena observed near the point of condensation of gas, where for example carbonic acid gas is undoubtedly brought into a state more nearly fulfilling Boyle's law by simply increasing the temperature at constant volume, and constant volume implicitly involves a constant mean value for r.

If the sign of $rd\mathrm{R}$ is positive, the amount of energy employed becomes potential; if, however, it is negative, it is an amount of potential energy previously stored which is now set free. The latter is undoubtedly the fact, which perhaps may appear more clearly when we say that when the gas is raised to such a temperature that it is completely dissociated, and all attractions between its molecules and atoms are completely destroyed, then all the potential energy, intermolecular and interatomic, which has been stored in the gas, will be set free and become kinetic. The term $rd\mathrm{R}$ expresses then the amount set free during the variation $d\tau$.

To obtain a conception of the relative magnitude of the terms in (35) let us take the hypothesis that R is a function of r, expressed by the equation $\mathrm{R}=fr^{-i}$,

$$\therefore \quad \mathrm{R}dr=fr^{-i}dr, \quad rd\mathrm{R}=-ifr^{-i}dr,$$

$$\therefore \quad rd\mathrm{R}=-i\mathrm{R}dr \quad . \quad . \quad . \quad . \quad (36).$$

from which it appears that when the attraction varies as the inverse i the power of the distance the energy set free is i times that which becomes potential. Different values of i, all greater than unity, have been proposed by various physicists to represent approximately the actual attraction.

On the other hypothesis suggested, namely, that R is a function of τ, such that R decrease as τ increases, it is evident that the term $rd\mathrm{R}$ is negative; and, as has been stated, that signifies that potential energy becomes kinetic by increasing temperature, although the relative amount cannot be found while the function, which R is of τ, is unknown.

The experiments of Thomson and Joule*, in making air and other gases expand freely in passing through a porous plug enable us to compare (34) and (35) with the results of experiment.

In the experiments now considered the variation of temperature is very small, and the gases obey the law of Boyle so closely that we are at liberty to assume $d(pv)=o$,

$$\therefore \text{ by (34)} \quad akd\tau=\tfrac{1}{2}\Sigma(\mathrm{R}dr+rd\mathrm{R}) \quad . \quad (37).$$

in which the second member depends upon mean intermolecular forces and distances. If we take the hypothesis of (36),

$$akd\tau=\tfrac{1}{2}\Sigma(1-i)r^{-i}dr) \quad . \quad . \quad . \quad (38).$$

in which, owing to the expansion of the gas, dr is positive. The experiments showed $d\tau$ to be invariably negative, thus agreeing with the previous statement that $(1-i)$ is probably negative.

The explanations of this experiment ordinarily given appear to be erroneous, although they likewise make $d\tau$ negative. The explanation assumes in effect that the total intermolecular work depends upon terms of the form $\mathrm{R}dr$, and further states that this work must be done at the expense of the sensible heat of the gas, thereby attempting to apply the principle of the conservation of energy to the case in hand. This is stated mathematically by the equation

$$\lambda kd\tau+\Sigma(\mathrm{R}dr)=0 \quad . \quad . \quad . \quad (39)$$

in which λ is a numerical coefficient. By comparing this with (37), when terms of the form $rd\mathrm{R}$ are neglected, it appears that the assumed equation (39) has its terms in $\mathrm{R}dr$ of the wrong sign. According to the theorem of the virial as expressed in (37), the experiment should have given $d\tau$ positive when terms in $rd\mathrm{R}$ vanish; but they do not so vanish, and (37) is the correct equation of the conservation of energy. The mistake in (39) is the assumption without proof that the internal work depends only upon terms of the form $\mathrm{R}dr$. The introduction of the principle of the conservation of energy gave the correct sign to $d\tau$, notwithstanding this assumption, for the

* Phil. Trans. R. Soc. Lond., 1853, 1854, 1862; or Wüllner's Experimental Physik, 3te Aufl. 1875, Bd. 3, S. 461.

form of the expression for the internal work is here of no consequence; but in other cases, where the principle of the conservation of energy could not be employed, the assumption that the internal work is expressed by terms of the form Rdr has led to erroneous results, contradicting experiment. This is notably true respecting the ratio of the specific heats discussed in Art. 8.

The well-known behavior of carbonic acid and other gases, as they approach the point of condensation, also shows that the terms in Rdr and $rd\mathrm{R}$ are of opposite sign, the latter being numerically the larger.

Let a unit of gas be compressed at constant temperature until its pressure is increased by an amount dp, then $d\tau=0$ and (34) becomes

$$-pdv=vdp=\tfrac{1}{3}\Sigma(Rdr+rd\mathrm{R}) \quad . \quad (40).$$

in which dv and dr are negative, because the gas is compressed; hence $-pdv$ and vdp are both positive, but Rdr is negative. Experiment shows that for a given increment dp, the decrement dv (and hence the term $-pdv$) is numerically larger than it should be by Boyle's law, *i.e.*, than when the intermolecular forces vanish. If the terms in Rdr were alone considered, they would, since they are negative, decrease rather than increase $-pdv$; hence it must be the terms in $rd\mathrm{R}$ which cause the observed numerical increase in dv.

In the case just treated the signs of both terms of $d(r\mathrm{R})$ are the opposite of those in the case of free expansion previously treated, because here compression takes place, and as is seen kinetic energy is set free by the terms in Rdr, but a relatively larger amount is transformed into potential energy by terms in $rd\mathrm{R}$.

Berthelot's* principle of the maximum work (*i.e.*, maximum heat) of chemical decomposition states that every chemical change, accomplished without the intervention of energy from without, tends to the production of the body, or the system of bodies, which set free the most heat.

This is announced as a law resting for its proof upon a vast array of experimental evidence, and is evidently of fundamental importance to the theory of chemistry. It seems, however, to depend theoretically upon the mechanical principles involved in (34), for at the instant of chemical decomposition the atoms must be regarded as separate bodies, obeying the laws of stationary motion.

In order to introduce the condition of no exchange of energy with external bodies, let $dv=0$. $\therefore\ pdv=0$, *i.e.*, there is no work of expansion against external pressure, and if vdp be small enough to be neglected, then (37) may be applied to this case. It appears, however, that the ordinary conditions of experiment from which the law was deduced would be better represented by supposing the pressure constant, *i. e.*, $dp=0$, $\therefore\ vdp=0$, in which case pdv the external work is, as appears from experimental evidence in general so inconsiderable as not to affect the correctness of the law, so that (37) may be considered in this case also to express approximately the relations to which it is desired to give expression.

Considering now the case when $dv=0$, the terms in (37) in Rdr are inconsiderable, since the mean distance of the atoms cannot be greatly changed when the total volume is constant. But the terms in $rd\mathrm{R}$ depend principally upon the increment $d\mathrm{R}$ of the attractions of the atoms in the final chemical arrangement over those in the initial. If the atoms in the final arrangement obey the greater attractions, then $rd\mathrm{R}$ is positive, and by (37) heat is set free, or the temperature of the body raised. Thus it appears that Berthelot's principle—that of several possible chemical decompositions in an isolated body, that one will occur which sets free the largest quantity of heat—follows as a consequence of the axiomatic principle that the more powerful interatomic attractions will always determine the rearrangement of the atoms into molecules.

8. Ratio of the Specific Heat at Constant Pressure to that at Constant Volume.—Let κk be the specific heat at constant pressure, and k that of constant volume, in foot lbs., then is their ratio κ a quantity which has been determined experimentally for a number of gases with considerable accuracy. Its theoretical value can be found as follows:

Let the temperature of a unit of mass of gas be augmented by $d\tau$ at constant pressure; let dh be the total amount of heat imparted,

$$\therefore \qquad dh=\kappa k dt.$$

* Essai de Méchanique Chémique, 1879, t. 2, p. 421.

Let ds be the increment of the sensible heat,

$$\therefore \quad ds = k d\tau.$$

Let dw be the external work performed,

$$\therefore \quad dw = p dv;$$

$$\therefore \text{ by (34) } dw = \tfrac{2}{3} a k d\tau - \tfrac{1}{3} \Sigma d(r\mathrm{R}).$$

Let dw_1 be the internal energy employed against intermolecular forces, then, as has been previously shown.

$$dw_1 = \Sigma d(r\mathrm{R}),$$

in which let $\Sigma d(r\mathrm{R}) = \beta k d\tau$, then β expresses how large a fraction dw_1 is of ds; and as has been shown in Art. 7, β is almost certainly negative for gases.

We may then write

$$dw + dw_1 = \tfrac{2}{3}(a + \beta) k d\tau.$$

Let dw_2 be the internal work done within the molecule against interatomic forces; and if we let

$$dw_2 = \gamma k d\tau,$$

then γ expresses what fraction dw_2 is ds. On the hypothesis that molecules are solids whose atoms stand at nearly invariable distances at all temperatures, but whose atomic attractions are decreased as the temperature rises, γ would consist almost entirely of terms in $rd\mathrm{R}$, and so be always negative.

But the total heat imparted being equal to the sum of its parts,

$$dh = ds + dw + dw_1 + dw_2 \quad . \quad . \quad . \quad (41).$$

Substitute the values of the various terms in (41), which have been just given, and divide by $k d\tau$,

$$\therefore \quad \kappa = 1 + \tfrac{2}{3} a + \tfrac{2}{3} \beta + \gamma \quad . \quad . \quad (42),$$

in which, as previously seen, β and γ are most probably negative but quite small.

Suppose for the moment that

$$\tfrac{2}{3} \beta + \gamma = 0 \quad . \quad . \quad . \quad . \quad (43),$$

i. e., the intermolecular and interatomic forces vanish, in which case, as shown previously, a has the values 1, $\frac{3}{5}$, $\frac{1}{2}$, according as the molecules are regarded as solid spheres, solids of revolution, or of other figure.

$$\therefore \text{ by (42), when } a = 1.0,\ 0.6,\ 0.5,$$
$$\text{then } \kappa = 1.67,\ 1.4,\ 1.33.$$

The vapor of mercury is perhaps the only gas regarded by chemists as non-atomic, and for it Kundt and Warburg* found $\kappa = 1.67$. In this case it would be difficult to conceive γ to have a value differing from zero; hence, also for this gas, $\beta = 0$ within the limits of experimental accuracy, *i.e.*, the forces between the molecules are insensible.

Atmospheric air is almost entirely composed of gases whose molecules are considered to be diatomic, and hence are supposed to be figures of revolution. A considerable number of determinations of κ for air by the most eminent do not differ much from the mean value 1.405, while the derived from Regnault's most accurate determination of the velocity of sound is 1.3945, which numbers are taken from Wüllner,* who treats this subject at length.

As to other diatomic gases, as H_2, O_2, N_2, CO, NO, HCl, the most probable experimental values of κ lie between 1.41 and 1.39, as given by Meyer,† and so far as these numbers show anything they confirm the truth of (43) for this kind of gases also.

All experimental determinations of κ for gases whose molecules consist of more than two atoms, lie, so far as known, between 1.33 and 1.25, and in some cases it would appear that the value of $\frac{2}{3}\beta + \gamma$ must be decidedly negative. This is the case in which from the number of atoms in the molecules we should expect the interatomic action to appear in more marked degree. It should be noticed that a is not itself independent of β and γ, and the form of its dependence upon them could be expressed were it possible to state what function R is of r and τ. But this fact will not apparently reverse the nature of the result we have obtained by supposing a constant. Especially is this the case if γ if numerically much larger than β as has been shown to be extremely probable, knowing as we do that the interatomic attractions are strong and the intermolecular feeble. These considerations add to the probability of the correctness of the hypothesis that molecules are hard, smooth bodies.

Watson, in his treatise,‡ regards it as impossible that β and γ can have negative values, and so finds the experimental

* Pogg. Ann., 1876; Bd. 157, S. 353.

* Lehrbuch der Experimental Physik, 3te Rufl. 1876 Bd. 3. S. 461.

† Kinetische Theorie der Gase, 1876, S. 90.

‡ Kinetic Theory of Gases, 1876, p. 38.

results inexplicable, while Boltzmann,* in his paper *Ueber die Nature der Gasmolecüle*, with whose conclusions mine are in general accordant, finds here the same contradiction as Watson between theory and experiment, due, as has been shown, to the omission of terms in rd R.

* Sitzb. d. Wein. Akad. 1876, Bd. 74.

ON THE SPEED AND POWER OF RAILWAY TRAINS.

By DR. H. SCHOEFFLER

Organ für die Fortschritte des Eisenbahnwesens. Selected Papers of the Institution of Civil Engineers.

THIS article gives the methods used on the Brunswick Railways for fixing the speed and size of trains, according to the gradients and curves on the line. The rules are approximate only, but sufficient for practice. The maximum speed in any given train may be determined from two points of view; (1) that of the power of the engine, (2) that of safety in running. For speeds below the lower of these two limits no rule can be given which does not include the fuel expended, and this embraces general considerations of economy, &c.

(1) *Speed at constant expenditure of Fuel.*—Let a be the train resistance, b the weight of the engine and tender, f the number of axles in the train (supposing an average load on each, say 5,000 kilogrammes), 1 in n the gradient, r the radius of the curve. The resistance of the train may be taken at $\frac{1}{360}$ of its weight; that of the engine as $\frac{1}{160}$. But to this must be added the resistances due to the air, &c., which are such that if b be expressed in the corresponding number of train axles, it will have a value varying from 4.5 b to 11 b. The resistance due to the curves may be expressed by adding to the gradient $\frac{3}{4r}$. From these figures the following formula is obtained, as representing the work done per hour by an engine at a speed of v kilometers per hour:

$$\left\{a+f+300\left(\frac{1}{1}+\frac{3}{4r}\right)(b+f)\right\}v.$$

From this expression the velocity v of a given load can be determined. Also the velocity v, or the time t, for a given length of track can be determined in terms of v_1 and t_1, the values for the same train on a straight horizontal road; or in terms of v_0, t_0, the values for the engine alone upon a straight horizontal road.

These formulæ show that the time per hour increases by a percentage of the time t_0, which percentage increases as the gradients are higher and the curves sharper. If it be determined to keep the time constant, a virtual length may be substituted for the real length of the line increased in the same proportion. This, throughout assumes that the consumption of fuel is constant, which is by no means a good assumption for practical working.

(2) *Maximum Speed with a given power of Engine.*—If the consumption of fuel is varied, the speed can evidently be increased up to the maximum power of the engine, or up to the limit of safety. The lower of these limits must be taken as the highest admissible speed. Now, for ordinary speeds the resistance of a train of x axles may be taken as proportional to

$$\left\{1+0.1\,v+1{,}000\left(\frac{1}{n}+\frac{3}{4r}\right)\right\}x,$$

where v is the velocity, $\frac{1}{n}$ the gradient, and r the radius of the curve. The resistance of the engine, taking its weight as equal to that on b axles, is given by

$$d+\left\{1+0.1\,v+1{,}000\left(\frac{1}{n}+\frac{3}{4r}\right)\right\}b,$$

where d is a number found from the resistance of the engine at the minimum speed. By adding these two formulæ together, an expression is obtained for the total resistance which the engine has to overcome at a given speed. The power of the engine being known, the maximum speed can then be calculated; or if the limit of speed is fixed, the number of axles it can draw at that limit can be

calculated. Examples of the calculation are given.

Taking v_1 as the velocity on a straight horizontal road, the following formula is obtained:

$$v = v_1 - 10{,}000\left(\frac{1}{n} + \frac{3}{4r}\right)$$

which shows how the speed is to be reduced to correspond with any given gradients and curves. If in the above formula $v=0$, the greatest number of axles are found which a train can move upon a given gradient.

(3) *Maximum Speed with reference to the conditions of the Road.*—The question of safety will often fix the maximum speed. In Germany, on gradients below 1 in 200, and curves of more than 1,000 meters radius, the maximum speed allowed is 90 kilometers per hour (55 miles) for express trains, 75 for ordinary trains (45 miles), and 45 for goods trains (27 miles); and this must be reduced on steeper gradients or sharper curves.

The distance x, in which a train will be brought to rest under the action of brakes, is given by the expression

$$\frac{v^2}{2g\left(\frac{1}{m} - \frac{1}{n}\right)},$$

where v is the speed, $\frac{1}{n}$ the falling gradient, and $\frac{1}{m}$ the retarding resistance, taken as a fraction of the weight of the train. Putting v equal to maximum speed, this gives the value for n on which this speed is allowable. The speed on any other gradient $\frac{1}{n}$ is given by

$$v\left(1 + \frac{m}{2n} - \frac{m}{2n'}\right).$$

The quantity m depends, of course, upon the brake arrangements. In Germany, passenger trains on gradients of 1 in 200 must have one-fifth of the wheels braked. If it be assumed that half the weight of the engine and tender is available for braking, and that the coefficient of friction is $\frac{1}{10}$, then $m=20$ for express trains, $m=30$ for passenger trains, and $m=40$ for goods trains. On these data a table is constructed, giving the maximum speed for the three classes on various gradients. The distances in which the train will be brought to rest are as follow:

Express trains, 600 to 700 meters (say 700 yards).

Ordinary trains, 700 to 800 meters (say 800 yards).

Goods trains, 300 to 400 meters (say 400 yards).

With regard to curves, it may be assumed that the centrifugal force is to be always the same; and a table constructed on this basis is given. It may be objected that the raising of the outer rail should be considered; but the German State regulations take no account of this, and it appears, in fact, that the relation between the speed and the radius of the curve is not altered thereby. The practical results are as follow: On gradients of 1 in 200 to 1 in 80, and curves of 1,100 to 900 yards radius, the speed must be reduced by one-eighth; on gradients of 1 in 80 to 1 in 40, and curves of 900 to 650 yards radius, by one-quarter; on curves of 650 to 450 yards radius, by three-eighths; on curves of 450 to 275 yards radius by one-half.

(4) *Maximum Speed with reference to the construction of the Engine.*—This point cannot be treated theoretically. In fixing the highest speed at which a given engine will run easily, account must be taken of all points in construction which tend to produce rolling, jolting, and other irregularties. Such points are: (*a*) the number of coupled wheels, whether 0, 4, or 6; (*b*) the overhang beyond the leading axle, which varies between 1.4 and 2.16 meters; (*c*) the overhang beyond the trailing axle, which varies between nothing and 2.46 meters; (*d*) the stroke, which varies between 0.53 and 0.66 meters; (*e*) the height of the center of gravity of the empty boiler above the rails, which varies between 1.6 and 1.9 meter; (*f*) the distance of the cylinder center-line from the engine center-line, which varies between 0.37 and 1.01 meter; (*g*) the distance of the middle of the cylinder in front of the center of gravity.

In all the above the evil increases as the dimensions increase. In those which follow, it increases as the dimensions diminish. These are: (*h*) the diameter

of the driving wheels, which varies between 1.1 and 1.83 meter; (*i*) the distance of the center-line of the driving-axle brasses from the engine center-line, which varies between 0.57 and 0.96 meter; (*k*) the wheel-base, which varies between 2.0 and 4.57 meters, and which belongs to this class, except on very sharp curves, where it would belong to the former class.

The author discusses these various elements, and arrives at the following formula (for dimensions in meters):

$$v = h(63 - 2.5\,a - 2.2\,c - 16.8\,f).$$

The influence of the quantities (*b*), (*d*), (*e*), (*g*), (*i*), and (*k*) is here neglected

From this formula the author has constructed a table, part of which is subjoined.

Class of engine.	Diameter of driving wheel. *n*	Number of axles coupled. *a*	Overhang behind. *c*	Distance of cylinder axis from loco-axis. *f*	Maximum speed. *v*	
	Meters.		Meters.	Meter.	Kilom.	Miles.
Prussian normal passenger engine.	1.730	2	1.585	0.940 (outside)	71.3	44.3
Prussian normal goods engine....	1.330	3	2.645	1.015 "	46.7	29.0
Lilleshall Co. express engine.....	2.134	.	0.700	0.380 (inside)	117.5	72.7
Great Northern railway express engine....	2.465		0.500	1.000 (outside)	111.2	69.0
London, Chatham, and Dover Railway express engine.....	1.980	2	0.219	0.356 (inside)	102.6	63.4

The general result is that no single formula can be given for the speed of trains, which must be regulated by the lowest of the three limits assigned by the power of the engines, by their construction, and by the conditions of the road. Certain general rules may be laid down. The maximum speed consistent with the curves and gradients should first be determined. If there is no limit in the choice of locomotives, this maximum speed can always be realized; but if there is no stringent necessity for it, it is better to reduce the speed, from considerations of economy in fuel, and in wear and tear. If, however, the class of engines to be used is fixed, their maximum speed, as determined both by their power and their construction, must be found; and the lower of these forms the limit of speed for that particular set of engines. This may, of course, be diminished as before from economical considerations.

THE GEOLOGICAL SURVEY OF ITALY.

From "Nature."

Amongst the numerous signs of renewed life which characterize United Italy, the rapid spread of scientific research must take important rank. In its old homes at the world-famous universities, science, not often entirely neglected, is now once more largely cultivated. New museums are springing up in many of the more important towns, and old ones are everywhere remodelled and enlarged.

Geology has its full share in this scientific revival, as a glance at the annual volumes of the *Geological Record* will show. The establishment of the Geological Society of Italy, to which we referred a fortnight back, will do much to encourage the study of this science.

The Geological Survey of Italy was established in the year 1868; since that date it has gradually developed, and has now accomplished some very important work. The Survey is at present a branch of the Corps of Mining Engineers, but we speak of the service in the phrase best known in England for similar organizations. Since 1870 twelve volumes of the *Bollettino del Ro. Comitato Geologico* have been published. These contain memoirs of various districts, often well illustrated, by members of the Survey or

by other workers whose essays are considered to be worth publishing at the public expense. Probably many memoirs of the latter class will in future find their way into the Geological Society's volume, and the *Bollettino* be more purely official.

The organization of the Survey is somewhat peculiar, and exhibits an amount of divided responsibility which can hardly conduce to its success. The service is partly under the control of a committee of eleven members and a secretary. Of this committee Prof. J. Meneghini of Pisa is president. Seven of the members are nominated by the King, chiefly from amongst the professors at the universities. These give their services gratuitously, only the actual traveling expenses being paid by the Government. The remainder are official members, and some of them are in other ways connected with the Survey.

Each member of the committee has a certain amount of influence in the control of the Survey within his own district; he is supposed to be consulted upon all questions relating to classification, naming of fossils, &c., but the surveyors are really responsible to the official chief of the Survey, M. G. Giordano. Three members of the Scientific Committee take the chief share in the direction. These are Prof. J. Meneghini for Tuscany and Rome; Prof. G. Cemmellaro of Palermo for Sicily; and Prof. J. Capellini of Bologna, whose advice and assistance is always freely at the disposal of the Survey. This dual government might have been desirable in the early stages of the Survey; but now that Italian geology has made such progress, the staff so well trained, and the work so far advanced, it will probably be desirable to reorganize the Survey upon its own basis, giving the sole responsibility to its own official chief.

The surveying staff is part of the Corps of Inspectors of Mines (Ingegneri delle Miniere), the Chief Inspector of which is also the Chief of the Survey. Italy is divided into eight mineral districts—Turin, Milan, Vicenza, Florence, Ancona, Naples, Caltanissetta, Iglesias (Sardinia). The Inspectors of Mines have duties very similar to those of officers holding a like position in England. They visit and report upon mines in cases of accidents, and when any important changes take place in the working of the mines they may be called on for advice. The engineers are chosen from students trained in one of the seven engineering schools of Italy (Turin, Milan, Padua, Bologna, Pisa, Naples, Palermo). They then go for two years to a foreign mining school (Berlin, Freiberg, London, or Paris). Those engineers who are to serve on the Geological Survey Staff receive additional instruction for this purpose. Till now this extra training has generally been obtained from the Geological Survey of England, so that we may regard the Italian Survey as in a certain sense related to our own. Of the officers thus trained in England we may mention M. Anselmo, L. Baldacci, L. Mazzetti, R. Travaglia, De Ferrari, and E. Cortese; to the last named of these we are indebted for much information here given.

The basis of every geological survey must be a good topographical map. The Austrians published a map of a great part of Italy on the scale of 1 : 75,000; this, however, is not satisfactory. An entirely new topographical survey is now in progress; commenced in Sicily in 1862, it is gradually advancing to the north. The general map of Italy is on the scale of 1 : 50,000, with contour lines at every 10 meters. The more important mineral districts are published on the scale of 1 : 25,000, with contour lines at every 5 meters; a very beautiful map of Rome and the surrounding country is now published on the larger scale, as also are Sicily and parts of the N. W. Apennines. There is a smaller map, on the scale of 1 : 100,000, with contours at every 50 meters.

The small sum voted annually by the Italian Parliament has hitherto been spent in surveying only, and none of the maps have yet been published. They were, however, all exhibited at the Geological Congress at Bologna. It is expected that a larger grant will now be made, and the Survey be placed on a more satisfactory footing. The Central Survey office at Rome, hitherto lodged in the Piazza S. Pietro in Vincoli, will shortly be transferred to the handsome buildings lately erected for the Ministry of Agriculture, Industry and Commerce.

The whole of the Island of Sicily has

been geologically surveyed on the largest scale. This district is of commercial importance from its great yield of sulphur, amounting to 250,000 tons per year. The entire map of Sicily on the scale of 1:50,000, was exhibited at Bologna; this is a very beautiful work, and will be of great service to all students of that most interesting region. The Apennines north of Pisa are also surveyed on the scale of 1:25,000. This district is of great importance from the marble quarries of Carrara, Massa, &c., the yield of which is 150,000 tons per year. Great uncertainty has long been felt as to the geological age of the Carrara marble; it contains no fossils, and its exact relation to adjacent formations has hitherto been doubtful. It has at various times been referred to many different geological horizons; but now the geological surveyors seem definitely to have fixed its position in the Trias. The mineral districts of Sardinia and the Campagna of Rome have also been surveyed on the scale of 1:25,000.

The complete geological survey of a country is a work of some time, and many years must elapse before that of Italy is finished and its maps published. In the meantime the Survey has done a most useful work in preparing a general geological map of Italy on the scale of 1:500,000. For this purpose all previously published information has been utilized; the geological notices scattered through various scientific journals, Italian and foreign, have been collected and arranged by M. Giordano and his colleagues. The numerous blanks have been filled up by special researches; and the result is a valuable and beautiful map, which will shortly be published. It was desired to issue a reduction of this on the scale of 1:1,000,000, but as no topographical map on this scale exists, a French map was adopted, engraved on the scale of 1:1,111,111 (or 1 decim. to a degree). This map was corrected where necessary, and was published in time for the meeting of the Congress at Bologna. The map is issued in two editions, one with hill-shading and one without. The only general map previously published was that of Collegno, in 1846, on the scale of 1:2,000,000. A glance at the two maps will show the immense advance which has been made in our knowledge of Italian geology since that date.

The map in question is colored in accordance with the scheme recommended by the Italian Map Committee of the International Geological Congress. The Italian Committee (like the English) prefer to retain some shades of red for the Trias. The Congress, however, chiefly influenced by the wishes of Germany, proposes to color this violet, as the natural base of the secondary series; the Jurassic beds being colored blue. The Italian Survey is desirous of adopting for its future maps the scheme of coloring upon which the Congress may decide. The Indian Survey also, being now about to publish a connected series of maps, wishes, if possible, to do the same. We have little doubt that the geological map of Europe, now being prepared by the Map Committee of the Congress, will be so drawn up and colored as to form a scheme of colors which can, with only small modifications, be adopted by all.

From the report of the United States Commissioner of Agriculture it appears that 2,500,000 packages of seeds have been distributed, and 260,000 copies of special reports printed by the department. The statistical division estimates the following as the yield of 1882:—Corn, 1,635,000,000 bushels; wheat, 400,000,000 bushels; oats, 470,000,000 bushels; barley, 45,000,000 bushels; rye, 20,000,000 bushels; and buckwheat, 12,000,000 bushels.

For waterproofing brick walls the following has been given. Dissolve soft parafine wax in benzoline spirit in the proportion of about 1 part of the former to 4 or 5 parts of the latter by weight. Into a tin or metallic keg place 1 gallon of benzoline spirit, then mix 1½ lb. or 2 lbs. wax, and when well hot pour into the spirit. Apply the solution to the walls whilst warm with a whitewash brush. To prevent the solution from chilling, it is best to place the tin in a pail of warm water, but on no account should the spirit be brought into the house, or near to a light, or a serious accident might occur.

HYDRAULIC TABLES BASED ON KUTTER'S FORMULA.

By P. J. FLYNN, C. E.

Contributed to VAN NOSTRAND'S ENGINEERING MAGAZINE.

THE tables given below are intended to facilitate the calculation of velocities, discharges, slopes and dimensions of sewers and other conduits, and their use will effect a great saving of time; as, for instance, instead of calculating the velocity and discharge by the use of a troublesome formula, the same result, practically, will be arrived at by taking the product of two factors given in the tables.

Kutter's formula is a complicated equation, and in its general form is:

$$v=c\sqrt{rs} \text{ in which}$$

$$c=\left\{\frac{41.6+\frac{1.811}{n}+\frac{.00281}{s}}{1+\left((41.6+\frac{.00281}{s})\times\frac{n}{\sqrt{r}}\right)}\right\}$$

In this and the following formulæ,

v=mean velocity in feet per second.

s=fall of water surface (h) in any distance (l) divided by that distance=$\frac{h}{l}$=sine of slope.

r=hydraulic mean depth=area of cross section of water divided by wetted perimeter=$\frac{a}{p}$.

d=diameter of circular channel.

a=area of cross section of water.

p=wetted perimeter.

Q=discharge in cubic feet per second.

n=the natural coefficient depending on the nature of the bed, that is, the lining of the channel over which the water flows, which throughout this article, and in the preparation of the tables, has been taken at .015.

Mr. J. C. Trautwine, in his *Engineer's Pocket Book*, states that, "In consideration of the rough character of sewer brickwork generally," he has taken n=.015 in Kutter's formula when he calculated the velocities in sewers.

Mr. R. Hering, in a paper read before the American Society of Civil Engineers in 1878 on the velocity and discharge of sewers, gave: "n=.015" for "foul and slightly tuberculated iron; cement and terra cotta pipes with imperfect joints, and in bad order; well dressed stonework and second-class brickwork." The tables do not apply to channels with smooth or plastered surfaces. They are intended to apply only to sewers, conduits and other channels whose surfaces exposed to the flow of water are of second-class brickwork, or have surfaces of other material equally rough, such, for instance, as those given above from Mr. Hering's paper.

The general form of Kutter's formula is:

$$v=c\sqrt{rs}=c\sqrt{r}\times\sqrt{s} \quad . \quad . \quad . \quad . \quad (1).$$

from which

$$c\sqrt{r}=\frac{v}{\sqrt{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

$$\sqrt{s}=\frac{v}{c\sqrt{r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

$$s=\left(\frac{v}{c\sqrt{r}}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

$$Q=av=ac\sqrt{r}\times\sqrt{s} \quad . \quad . \quad . \quad . \quad (5).$$

from which

$$a=\frac{Q}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

$$ac\sqrt{r}=\frac{Q}{\sqrt{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

$$\sqrt{s}=\frac{Q}{ac\sqrt{r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

$$s=\left(\frac{Q}{ac\sqrt{r}}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

The values of $c\sqrt{r}$ and $ac\sqrt{r}$ for 124 diameters are given in table 1, and the values of $\sqrt{s}$ for 557 slopes are given in table 2. It will then be seen that a large range of channels numbering 69068 are included in these tables. The velocity is found by the product of two factors $c\sqrt{r}$ and $\sqrt{s}$, and in a similar way the discharge is found by the product of the two factors $ac\sqrt{r}$ $\sqrt{s}$.

In Kutter's formula given above the value of c is found from an equation involving the value of r, n and s, so that any change in the value of s would cause a change in the value of c, but as the influence of s on the value of c is not very marked in such slopes as are usually adopted for sewers and conduits, the value of the coefficient has been calculated for one slope, that of 1 in 1000 or $s=.001$. This value of the coefficient is *practically constant* for all values of s with a steeper slope than 1 in 1000, and as sewers are generally designed with steeper slopes than 1 in 1000, the tables are well adapted to facilitate the calculations. For flatter slopes than 1 in 1000 up to even 2 feet per mile, or 1 in 2640, the tables give results showing a maximum error in the case of a sewer 2 feet in diameter of less than 2 per cent., and in the case of a sewer 8 feet in diameter less than $\frac{1}{2}$ per cent.; therefore, for all practical purposes, the tables are sufficiently accurate.

The hydraulic mean depth of a cylindrical conduit flowing full is equal to one-fourth of the diameter.

The *mean velocity* in circular sewers and conduits is the same when running half full as when running full.

Application and Use of the Tables.

To find the mean velocity in feet per second and the discharge in cubic feet per second.

Example 1.—A circular brick sewer has a diameter of 3 feet and a fall of 1 in 500. What is its mean velocity in feet per second and also its discharge in cubic feet per second?

By formula (1) $v=c\sqrt{r}\times\sqrt{s}$.

In column 4 of table 1 and opposite 3 feet diameter the value $c\sqrt{r}$ is found equal to 80.77, and in table 2 opposite 1 in 500 the value of $\sqrt{s}$ is found equal to .044721; substituting these values in equation, we have:

$v=80.77\times.044721$

$=3.61$ feet per second the mean velocity.

By formula (5) $Q=av=a\times3.61$, but by table 2 the area of a sewer 3 feet in diameter=7.068; substitute this value in equation and

$Q=7.068\times3.61$

$=25.52$, the discharge in cubic feet per second.

Again, as a check,

By formula (5) $Q=ac\sqrt{r}\times\sqrt{s}$.

In column 4 of table, and opposite 3 feet diameter the value of $ac\sqrt{r}$ is given as 570.9, substituting this value and also the value of $\sqrt{s}$, as found above, in equation we have

$$Q=570.9\times.044721=25.53$$

cubic feet per second the discharge, which is the same as already found above.

Example 2.—*To find the diameter.*—(*d*). The grade (s) of a sewer is to be 1 in 480, and its mean velocity (v) 4 feet per second. What is the required diameter? By formula (2),

$$c\sqrt{r}=\frac{v}{\sqrt{s}}$$

In column 3 of table 2 we find for a slope of 1 in 480 that $\sqrt{s}$ is equal to .045644. Substitute this in equation, and also the value of v already given, and

$$c\sqrt{r}=\frac{4}{.045644}=87.63.$$

Now look in column 4 of table 1 for the nearest value of $c\sqrt{s}$ to this which we find to be 87.15, opposite 3 feet 4 inches in diameter, which is the diameter required.

Example 3.—*To find the grade of Sewer.*—A sewer 2 feet 6 inches diameter is to have a velocity when running full or half full of not more than $3\frac{1}{2}$ feet a second. What should its grade be?

By formula (3) $\sqrt{s}=\frac{v}{c\sqrt{r}}$.

In column 4 of table 1 find opposite the diameter 2 feet 6 inches that $c\sqrt{r}$ is equal to 70.74. Substitute this value and also the value of v already given in equation, and $\sqrt{s}=\frac{3.5}{70.74}=.049477$. Now look out the nearest the value of $\sqrt{s}$ to this in column 3 of table 2, which we find to be .049507, opposite a slope of 1 in 408, which is the required grade.

To find the grade of sewer when the grade is not given in Table 2.

Example 4.—A sewer having a diameter of 1 foot 9 inches is to have a velocity of $2\frac{1}{4}$ feet per second. What is its required grade?

By formula (3) $\sqrt{s}=\frac{v}{c\sqrt{r}}$. Look out

the value of $c\sqrt{r}$ for 1 foot 9 inches diameter in Table 1, and it will be found to be 54.29. Substitute this value and also the value of v already given in equation, and

$$\sqrt{s}=\frac{2.25}{54.29}=.041444.$$

On looking for this value on Table 1, we find that table does not extend to a flatter slope than that whose $\sqrt{s}=.042254$. Therefore square each side of the equation:

$\sqrt{s}=.041444$ and we get
$s=.001717$

and $\frac{1}{.001717}=582$, therefore the slope is 1 in 582.

To find the diameter (d).

Example 5.—A sewer is to discharge 9 cubic per second and its grade is to be 1 in 200. What is its diameter to be?

By formula (7) $ac\sqrt{r}=\frac{Q}{\sqrt{s}}$. In the third column of table 2 and opposite 1 in 200 the value of $\sqrt{s}$ is found to be .070710. Substitute this value and the discharge already given in equation, and we have $ac\sqrt{r}=\frac{9}{.070710}=127.28$. In column 5 of Table 1, the value of $ac\sqrt{r}$ nearest to this we find to be 130.58, opposite to which is the diameter of 1 foot 9 inches, which is the diameter required.

To find the grade or slope of sewer. (s).

Example 6.—A sewer 6 feet in diameter is required to discharge 180 cubic feet of water per second. What should be its slope?

By formula (8) $\sqrt{s}=\frac{Q}{ac\sqrt{r}}$. In column 5 of Table 1 and opposite 6 feet in diameter the valve of $ac\sqrt{r}$ is found equal to 3702.3. Substitute this and also the value of Q in equation, and we have

$$\sqrt{s}=\frac{180}{3702.3}=.048618.$$

Now in column 4 of Table 2 look out the number nearest to this, which will be found to be .048621 opposite a slope 1 in 423, therefore the required grade is 1 in 423.

To find diameters in a series of sewers with increasing discharge.

Example 7.—A circular sewer has for 500 feet in length to discharge 10 cubic feet per second, then for 600 feet more has to discharge 12 cubic feet feet per second, and again for 700 feet, farther on 15 cubic feet per second. The total fall available is 5 feet. What is the required diameter and fall of each section? The total length is 1800 feet and $\frac{5}{1800}=.002777=s$ and $\sqrt{.002777}=.052705=\sqrt{s}$.

By formula (7) $ac\sqrt{r}=\frac{Q}{\sqrt{s}}$.

In this equation substitute values of Q and s for each section and find the corresponding diameters, which will be the diameters required.

$ac\sqrt{r}=\frac{10}{.052705}=189.7$ } Opposite which in Table 1 is { diam. 2′−0″

$ac\sqrt{r}=\frac{12}{.052705}=227.7$ } { diam. 2′−2″

$ac\sqrt{r}=\frac{15}{.052705}=284.6$ } { diam. 2′−4″

Now $s=\frac{h}{l}$ ∴ $h=sl$, therefore the

Fall of first section $=sl=.002777$
$\times 500 \ldots\ldots = 1.39$ ft.
Fall of second section $=sl=.002777$
$\times 600 \ldots\ldots = 1.67$ "
Fall of third section $=sl=.002777$
$\times 700 \ldots\ldots = 1.95$ "

Total fall $\ldots\ldots$ $\ldots\ldots 5.00$ ft.

We have, therefore,

1st	section, diameter	2′−0″,	fall 1.39 ft.
2d	" "	2′−2″	" 1.67 "
3d	" "	2′−4″	" 1.94 "

To find velocity and discharge of trapezoidal channel.

Example 8.—A trapezoidal channel lined with brickwork, 6 feet wide at bottom and with side slopes of 1 to 1, has 2 feet in depth of water and a grade of 1 in 160. What is its velocity and discharge per second?

Area $(a)=\frac{6+10}{2}\times 2=16$ square feet.

Wetted perimeter $(p)=2\times\sqrt{2^2\times 2^2}+6=11.66$ feet.

∴ Hydraulic mean depth

$$(r)=\frac{d}{p}=\frac{16}{11.66}=1.372.$$

In column 3 of Table 1 look out the

nearest value of r to this which we find to be 1.375, and corresponding to this we find $c\sqrt{r}$ equal to 123.5. In Table 2 for a slope of 1 in 160 the value of $\sqrt{s}$ is found to be =.079057.

Now by formula (1) $v=c\sqrt{r}\times\sqrt{s}$ and " " " (5) $Q=av$ substituting the values above found of the factors, then $v=123.5\times079057=9.76$ feet per second and $Q=16\times9.76=156.2$ cubic feet per second, therefore the mean velocity is equal to 9.76 feet per second and the discharge equal to 156.2 cubic feet per second.

To find the dimensions of a circular sewer to replace a rectangular brick channel.

Example 9. An open brick channel 5 feet wide at bottom, with vertical sides, has a depth of water in floods of 3 feet and a slope of 1 in 520. It is intended to substitute for it a circular sewer whose mean velocity flowing full shall be about 5 ft. per second. What should be the diameter and grade of the new circular sewer flowing full?

In Table 2 the $\sqrt{s}$ for a grade of 1 in 520=.043853.

Area of rectangular channel (a)=5×3 =15 sq. ft. Wetted perimeter (p)=3+ 5+3=11 feet. ∴ Hydraulic mean depth (r)=$\frac{d}{p}=\frac{15}{11}$=1.364. In Table 1 find corresponding to this hydraulic mean depth the nearest $c\sqrt{r}$, which is 123.5.

By formula (5) $Q=a\times c\sqrt{r}\times\sqrt{s}$ substitute the values found above, of the factors in right hand side of equation, and $Q=15\times123.5\times.043853=81.24$ cubic feet per second, the discharge from the rectangular channel.

We have now to find the diameter and grade of a circular sewer to convey this quantity of water with a velocity not greater than 5 feet per second.

By formula (6) $a=\frac{Q}{v}$ substitute values $a=\frac{81.26}{5}=16.252$ square feet=area of circular sewer. In column 2 of Table 1 we find the area nearest in value to this =16.499, and the corresponding diameter equal to 4 feet 7 inches, and at the same time find the value of the corresponding $ac\sqrt{r}$ which is 1796.5.

By formula (8) $\sqrt{s}=\frac{Q}{ac\sqrt{r}}$ substitute values of Q and $ac\sqrt{r}$ found above and

$$\sqrt{s}=\frac{81.26}{1796.5}=045232.$$

In Table 2 we find the grade corresonding to this equal to 1 in 489, therefore the diameter of circular sewer is 4 feet 7 inches, and the grade 1 in 489.

TABLE I.—GIVING VALUES OF a AND r AND ALSO THE FACTORS $c\sqrt{r}$ AND ALSO $ac\sqrt{r}$.

These factors are to be used only where the value of N, that is the coefficient of roughness of lining of channel=.015 as in second class or rough-faced brickwork.

$v=c\sqrt{r}\times\sqrt{s}$. $Q=av=ac\sqrt{r}\times\sqrt{s}$.

d = diameter in ft.	in.	a = area in square ft.	r=hydraulic mean depth.	$c\sqrt{r}$	$ac\sqrt{r}$	d = diameter in ft.	in.	a = area in square ft.	r=hydraulic mean depth.	$c\sqrt{r}$	$ac\sqrt{r}$
0	5	0.136	0.104	17.36	2.3615	1	6	1.767	0.375	48.38	85.496
0	6	0.196	0.125	20.21	3.9604	1	7	1.969	0.396	50.40	99.242
0	7	0.267	0.146	22.95	6.1268	1	8	2.182	0.417	52.45	114.46
0	8	0.349	0.167	25.56	8.9194	1	9	2.405	0.437	54.29	130.58
0	9	0.442	0.187	28.10	12.421	1	10	2.640	0.458	56.29	148.61
0	10	0.545	0.208	30.52	16.633	1	11	2.885	0.479	58.20	167.90
0	11	0.660	0.229	33.03	21.798	2	0	3.142	0.500	60.08	188 77
1	0	0.785	0.250	35.40	27.803	2	1	3.409	0 521	61.95	211.20
1	1	0.922	0.271	37.60	34.664	2	2	3.687	0.542	63 72	234.94
1	2	1 069	0.292	39.85	42.602	2	3	3.976	0.562	65.51	260.47
1	3	1.227	0.312	42.05	51.600	2	4	4.276	0.583	67.32	287.87
1	4	1.396	0.333	44.19	61.685	2	5	4.587	0.604	69.02	316.59
1	5	1.576	0.354	46.36	73.066	2	6	4.909	0.625	70.74	347.28

TABLE I.—GIVING VALUES OF a AND r AND ALSO THE FACTORS $c\sqrt{r}$ AND $ac\sqrt{r}$ FOR CORRESPONDING DIAMETERS IN COLUMN I.

$v = c\sqrt{r} \times \sqrt{s}$. $Q = av = ac\sqrt{r} \times \sqrt{s}$.

d = diameter in ft.	in.	a = area in square ft.	r = hydraulic mean depth.	$c\sqrt{r}$	$ac\sqrt{r}$	d = diameter in ft.	in.	a = area in square ft.	r = hydraulic mean depth.	$c\sqrt{r}$	$ac\sqrt{r}$
2	7	5.241	0.646	72.59	380.46	8	0	50.266	2.000	158.7	7978.3
2	8	5.585	0.667	74.27	414.81	8	3	53.456	2.062	162.0	8658.8
2	9	5.939	0.687	75.98	451.23	8	6	56.745	2.125	165.3	9377.9
2	10	6.305	0.708	77.56	488.99	8	9	60.132	2.187	168.4	10128
2	11	6.681	0.729	79.16	528.85	9	0	63.617	2.250	171.6	10917
3	0	7.068	0.750	80.77	570.90	9	3	67.201	2.312	174.7	11740
3	1	7.466	0.771	82.39	615.14	9	6	70.882	2.375	177.7	12594
3	2	7.875	0.792	84.03	661.77	9	9	74.662	2.437	180.7	13489
3	3	8.295	0.812	85.54	709.56	10	0	78.540	2.500	183.7	14426
3	4	8.726	0.833	87.15	760.44	10	3	82.516	2.562	186.7	15406
3	5	9.169	0.854	88.61	812.38	10	6	86.590	2.625	189.5	16412
3	6	9.621	0.875	90.11	866.91	10	9	90.763	2.687	192.4	17462
3	7	10.084	0.896	91.60	923.70	11	0	95.033	2.750	195.2	18555
3	8	10.559	0.917	93.11	983.11	11	3	99.402	2.812	198.1	19694
3	9	11.044	0.937	94.62	1045.0	11	6	103.87	2.875	201.0	20879
3	10	11.541	0.958	96.15	1109.6	11	9	108.43	2.937	203.7	22093
3	11	12.048	0.979	97.55	1175.2	12	0	113.10	3.000	206.5	23352
4	0	12.566	1.000	99.10	1245.3	12	3	117.86	3.062	209.2	24658
4	1	13.096	1.021	100.5	1315.8	12	6	122.72	3.125	212.0	26012
4	2	13.635	1.042	102.0	1390.8	12	9	127.68	3.187	214.6	27399
4	3	14.186	1.062	103.4	1466.7	13	0	132.73	3.250	217.4	28850
4	4	14.748	1.083	104.8	1545.7	13	3	137.88	3.312	220.0	30330
4	5	15.321	1.104	106.2	1627.0	13	6	143.14	3.375	222.6	31860
4	6	15.904	1.125	107.6	1711.4	13	9	148.49	3.437	225.2	33441
4	7	16.499	1.146	108.9	1796.5	14	0	153.94	3.500	227.8	35073
4	8	17.104	1.167	110.3	1886.8	14	3	159.48	3.562	230.0	36736
4	9	17.721	1.187	111.6	1977.7	14	6	165.13	3.625	232.9	38454
4	10	18.348	1.208	113.0	2074.1	14	9	170.87	3.687	235.4	40221
4	11	18.986	1.229	114.4	2172.9	15	0	176.72	3.750	237.9	42040
5	0	19.635	1.250	115.7	2272.7	15	3	182.65	3.812	240.5	43931
5	1	20.295	1.271	117.1	2376.7	15	6	188.69	3.875	242.8	45820
5	2	20.966	1.292	118.4	2482.0	15	9	194.83	3.937	245.3	47792
5	3	21.648	1.312	119.7	2590.5	16	0	201.06	4.000	247.8	49823
5	4	22.340	1.333	121.0	2702.1	16	3	207.40	4.062	250.3	51904
5	5	23.044	1.354	122.2	2816.7	16	6	213.83	4.125	252.7	54056
5	6	23.758	1.375	123.5	2934.8	16	9	220.35	4.187	254.9	56171
5	7	24.484	1.396	124.8	3056.4	17	0	226.98	4.250	257.2	58387
5	8	25.220	1.417	126.0	3177.3	17	3	233.71	4.312	259.7	60700
5	9	25.967	1.437	127.3	3305.6	17	6	240.53	4.375	261.9	62999
5	10	26.725	1.558	128.6	3436.3	17	9	247.45	4.437	264.4	65428
5	11	27.494	1.479	129.7	3566.6	18	0	254.47	4.500	266.6	67839
6	0	28.274	1.500	131.0	3702.3	18	3	261.59	4.562	268.9	70346
6	3	30.680	1.562	134.6	4130.3	18	6	268.80	4.625	271.3	72916
6	6	33.183	1.625	138.3	4588.3	18	9	276.12	4.687	273.5	75507
6	9	35.785	1.687	141.8	5074.7	19	0	283.53	4.750	275.8	78201
7	0	38.485	1.750	145.3	5591.6	19	3	291.04	4.812	278.0	80216
7	3	41.283	1.812	148.7	6136.8	19	6	298.65	4.875	280.2	83686
7	6	44.179	1.875	152.0	6717.0	19	9	306.36	4.937	282.4	86526
7	9	47.173	1.937	155.5	7333.5	20	0	314.16	5.000	284.6	89423

TABLE 2—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=fall of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
560	.001785714	.042258	510	.001960784	.044281
559	.001788909	.042295	509	.001964637	.044324
558	.001792115	.042333	508	.001968504	.044368
557	.001795332	.042371	507	.001972387	.044412
556	.001798561	.042410	506	.001976285	.044455
555	.001801802	.042448	505	.001980198	.044499
554	.001805054	.042486	504	.001984127	.044544
553	.001808318	.042524	503	.001988072	.044588
552	.001811594	.042563	502	.001992032	.044632
551	.001814882	.042601	501	.001996008	.044677
550	.001818182	.042640	500	.002000000	.044721
549	.001821494	.042679	499	.002004008	.044766
548	.001824817	.042718	498	.002008032	.044811
547	.001828154	.042757	497	.002012072	.044856
546	.001831502	.042796	396	.002016128	.044901
545	.001834862	.042835	495	.002020202	.044947
544	.001838235	.042874	494	.002024291	.044992
543	.001841621	.042914	493	.002028398	.045037
542	.001845018	.042953	492	.002032520	.045085
541	.001848429	.042993	491	.002036660	.045129
540	.001851852	.043033	490	.002040816	.045175
539	.001855288	.043073	489	.002044990	.045222
538	.001858736	.043113	488	.002049180	.045268
537	.001862197	.043153	487	.002053388	.045314
536	.001865672	.043193	486	.092057613	.045361
535	.001869159	.043234	485	.002061856	.045407
534	.001872659	.043274	484	.002066116	.045454
533	.001876173	.044315	483	.002070393	.045502
532	.001879699	.043355	482	.002074689	.045549
531	.001883239	.043396	481	.002079002	.045596
530	.001886792	.043437	480	.002083333	.045644
529	.001890359	.043478	479	.002087683	.045691
528	.001893939	.043519	478	.002092050	.045739
527	.001897533	.043561	477	.002096436	.045787
626	.001901141	.043602	476	.002100840	045835
525	.001904762	.043644	475	.002105263	.045883
524	.001908397	.043685	474	.002109705	.045932
523	.001912046	.043727	473	.002114165	.045980
422	.001915709	.043769	472	.002118644	.046029
521	.001919386	.043811	471	.002123142	.046077
520	.001923077	.043853	470	.002127660	.046126
519	.001926782	.043895	469	.002132196	.046176
518	.008930502	.043937	468	.001136752	.046225
517	.001944246	.043979	467	.002141328	.046274
516	.001937984	.044022	466	.002145923	.046324
515	.001941748	.044065	465	.002150538	.046374
514	.001945525	.044108	464	.002155172	.046424
513	.001949318	.044151	463	.002159827	.046474
512	.001953125	.044194	462	.002164502	.046524
511	.001956947	.044237	461	.002169197	.046575

TABLE 2 *(Continued)*.—GIVING VALUES OF s AND $\sqrt{s}$.

h = sine of slope = fall of water surface (h) in any distance (l), divided by that distance = $\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
460	.002173913	.046625	410	.002439024	.049387
459	.002178649	.046676	409	.002444988	.049447
458	.002183406	.046726	408	.002450980	.049507
457	.002188184	.046778	407	.002457002	.049568
456	.002192982	.046829	406	.002463054	.049629
455	.002197802	.046880	405	.002469136	.049690
454	.002202643	.046932	404	.002475248	.049752
453	.002207506	.046984	403	002481390	.049814
452	.002212389	.047036	402	.002487562	.049876
451	.002217295	.047088	401	.002493766	.049938
450	.002222222	.047140	400	.002500000	.050000
449	.002227194	.047193	399	.002506266	.050062
448	.002232143	.047245	398	.002512563	.050125
447	.002237136	.047298	397	.002518892	.050188
446	.002242152	.047351	396	.002525253	.050252
445	.002247191	.047404	395	.002531646	.050315
444	.002252252	.047458	394	.002538071	.050379
443	.002257336	.047511	393	.002544529	.050443
442	.002262443	.047565	392	.002551020	.050507
441	.002267574	.047619	391	.002557545	.050572
440	.002272727	.047673	390	.002564103	.050637
439	.002277904	.047728	389	.002570694	.050702
438	.002283105	.047782	388	.002577320	.050767
437	.002288330	.047836	387	.002583979	.050833
436	.002293578	.047891	386	.002590674	.050899
435	.002298851	.047946	385	.002597403	.050965
434	.002304147	.048001	384	.002604167	.051031
433	.002309469	.048057	383	.002610966	.051097
432	.002314815	.048113	382	.002617801	.051164
431	.002320186	.048168	381	.002624672	.051231
430	.002325581	.048224	380	.002631579	.051299
429	.002331002	.048280	379	.002638522	.051366
428	.002336449	.048337	378	.002645503	.051434
427	.002341920	.048393	377	.002652520	.051502
426	.002347418	.048450	376	.002659574	.051571
425	.002352941	.048507	375	.002666667	.051640
424	.002358491	.048564	374	.002673797	.051709
423	.002364066	.048621	373	.002680965	.051778
422	.002369668	.048679	372	.002688172	.051847
421	.002375297	.048737	371	.002695418	.051917
420	.002380952	.048795	370	.002702703	.051988
419	.002386635	.048853	369	.002710027	.052060
418	.002392344	.048911	368	.002717391	.052129
417	.002398082	.048970	367	.002724796	.052199
416	.002403846	.049029	366	.002732240	.052270
415	.002409639	.049088	365	.002739726	.052342
414	.002415459	.049147	364	.002747253	.052414
413	.002421308	.049207	363	.002754821	.052486
412	.002427184	.049266	362	.002762431	.052559
411	.002433090	.049326	361	.002770083	.052632

TABLE 2 *(Continued)*—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=full of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=side of slope.	$\sqrt{s}$.
360	.002777778	.052705	310	.003225806	.056796
359	.002785515	.052778	309	.003236246	.056888
358	.002793296	.052851	308	.003246753	.056980
357	.002801120	.052925	307	.003257329	.057073
356	.002808989	.052999	306	.003267974	.057166
355	.002816901	.053074	305	.003278689	.057260
354	.002824859	.053149	304	.003289474	.057354
353	.002832861	.053224	303	.003300330	.057449
352	.002840909	.053300	302	.003311258	.057544
351	.002849003	.053376	301	.003322259	.057639
350	.002857143	.053452	300	.003333333	.057735
349	.002865330	.053529	299	.003344482	.057831
348	.002873563	.053606	298	.003355705	.057929
347	.002881844	.053683	297	.003367003	.058026
346	.002890171	.053760	296	.003378378	.058124
345	.002898551	.053838	295	.003389831	.058222
344	.002906977	.053916	294	.003401361	.058321
343	.002915452	.053995	293	.003412969	.058420
342	.002923977	.054074	292	.003424658	.058520
341	.002932551	.054153	291	.003436426	.058621
340	.002941176	.054232	290	.003448276	.058722
339	.002949853	.054312	289	.003460208	.058824
338	.002958580	.054393	288	.003472222	.058926
337	.002967359	.054474	287	.003484321	.059028
336	.002976190	.054555	286	.003496503	.059131
335	.002985075	.054636	285	.003508772	.059235
334	.002994012	.054717	284	.003521127	.059339
333	.003003003	.054799	283	.003533569	.059444
332	.003012048	.054882	282	.008546099	.059549
331	.003021148	.054965	281	.003558719	.059655
330	.003030303	.055048	280	.003571429	.059761
329	.003039514	.055132	279	.003584229	.059868
328	.003048780	.055216	278	.003597122	.059976
327	.003058104	.055300	277	.003610108	.060084
326	.003067485	.055385	276	.003623188	.060193
325	.003076923	.055470	275	.003636364	.060302
324	.003086420	.055556	274	.003649635	.060412
323	.003095975	.055641	273	.003663004	.060523
322	.003105590	.055728	272	.003676471	.060634
321	.003115265	.055815	271	.003690037	.060746
320	.003125000	.055902	270	.003703704	.060858
319	.003134796	.055989	269	.003717472	.060971
318	.003144654	.056077	268	.003731343	.061085
317	.003154574	.056165	267	.003745319	.061199
316	.003164557	.056254	266	.003759398	.061314
315	.003174603	.056344	265	.003773585	.061430
314	.003184713	.056433	264	.003787879	.061546
313	.003194888	.056523	263	.003802281	.061662
312	.003205128	.056614	262	.003816794	.061780
311	.003215434	.056705	261	.003831418	.061899

TABLE 2 *(Continued)*—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=fall of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
260	.003846154	.062018	210	.004761905	.069007
259	.003861004	.062137	209	.004784689	.069172
258	.003875969	.062257	208	.004807692	.069338
257	.003891051	.062378	207	.004830918	.069505
256	.003906250	.062500	206	.004854369	.069673
255	.003921569	.062622	205	.004878049	.069843
254	.003937008	.062746	204	.004901961	.070014
253	.003952569	.062870	203	.004926108	.070186
252	.003968254	.062994	202	.004950495	.070359
251	.003984064	.063119	201	.004975124	.070534
250	.004000000	.063246	200	.005000000	.070710
249	.004016064	.063372	199	.005025126	.070888
248	.004032258	.063500	198	.005050505	.071067
247	.004048583	.063629	197	.005076142	.071247
246	.004065041	.063758	196	.005102041	.071429
245	.004081623	.063888	195	.005128205	.071612
244	.004098361	.064018	194	.005154639	.071796
243	.004115226	.064150	193	.005181347	.071982
242	.004132231	.064283	192	.005208333	.072169
241	.004149378	.064416	191	.005235602	.072357
240	.004166667	.064549	190	.005263158	.072548
239	.004184100	.064685	189	.005291005	.072739
238	.004201681	.064820	188	.005319149	.072932
237	.004219409	.064957	187	.005347594	.073127
236	.004237288	.065094	186	.005376344	.073324
235	.004255319	.065233	185	.005405405	.073521
234	.004273504	.065372	184	.005434783	.073721
233	.004291845	.065512	183	.005464481	.073922
232	.004310345	.065653	182	.005494505	.074125
231	.004329004	.065795	181	.005524862	.074329
230	.004347826	.065938	180	.005555556	.074536
229	.004366812	.066082	179	.005586592	.074744
228	.004385965	.066227	178	.005617978	.074953
227	.004405286	.066372	177	.005649718	.075164
226	.004424779	.066519	176	.005681818	.075378
225	.004444444	.066667	175	.005714286	.075593
224	.004464286	.066815	174	.005747126	.075810
223	.004484305	.066965	173	.005780347	.076029
222	.004504505	.067116	172	.005813953	.076249
221	.004524887	.067267	171	.005847953	.076472
220	.004545455	.067419	170	.005882353	.076697
219	.004566210	.067574	169	.005917160	.076923
218	.004587156	.067729	168	.005952381	.077152
217	.004608295	.067885	167	.005988024	.077382
216	.004629630	.068041	166	.006024096	.077615
215	.004651163	.068199	165	.006060606	.077850
214	.004672897	.068358	164	.006097561	.078087
213	.004694836	.068519	163	.006134969	.078326
212	.004716981	.068680	162	.006172840	.078568
211	.004739336	.068843	161	.006211180	.078811

TABLE 2 *(Continued)*—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=fall of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
160	.006250000	.079057	110	.009090909	.095346
159	.006289308	.079305	109	.009174312	.095783
158	.006329114	.079556	108	.009259259	.096225
157	.006369427	.079809	107	.009345794	.096674
156	.006410256	.080065	106	.009433962	.097129
155	.006451613	.080322	105	.009523810	.097590
154	.006493506	.080582	104	.009615385	.098058
153	.006535948	.080845	103	.009708738	.098533
152	.006578947	.081111	102	.009803922	.099015
151	.006622517	.081379	101	.009900990	.099504
150	.006666667	.081650	100	.010000000	.100000
149	.006711409	.081923	99	.010101010	.100504
148	.006756757	.082199	98	.010204082	.101015
147	.006802721	.082479	97	.010309278	.101535
146	.006849315	.082760	96	.010416667	.102062
145	.006896552	.083046	95	.010526316	.102598
144	.006944444	.083333	94	.010638298	.103142
143	.006993007	.083624	93	.010752688	.103695
142	.007042254	.083918	92	.010869565	.104257
141	.007092199	.084215	91	.010989011	.104828
140	.007142857	.084516	90	.011111111	.105409
139	.007194245	.084819	89	.011235955	.106000
138	.007246377	.085126	88	.011363636	.106600
137	.007299270	.085436	87	.011494253	.107211
136	.007352941	.085749	86	.011627907	.107833
135	.007407407	.086066	85	.011764706	.108465
134	.007462687	.086387	84	.011904762	.109109
133	.007518797	.086711	83	.012048193	.109764
132	.007575758	.087039	82	.012195122	.110431
131	.007633588	.087370	81	.012345679	.111111
130	.007692308	.087706	80	.012500000	.111803
129	.007751938	.088045	79	.012658228	.112509
128	.007812500	.088388	78	.012820513	.113228
127	.007874016	.088736	77	.012987013	.113961
126	.007936508	.089087	76	.013157895	.114708
125	.008000000	.089442	75	.013333333	.115470
124	.008064516	.089803	74	.013513514	.116248
123	.008130081	.090167	73	.013698630	.117041
122	.008196721	.090536	72	.013888889	.117851
121	.008264463	.090909	71	.014084507	.118678
120	.008333333	.091287	70	.014285714	.119524
119	.008403361	.091669	69	.014492754	.120386
118	.008474576	.092057	68	.014705882	.121286
117	.008547009	.092450	67	.014925353	.122169
116	.008620690	.092848	66	.015151515	.123091
115	.008695652	.093250	65	.015384615	.124035
114	.008771930	.093659	64	.015625000	.125000
113	.008849558	.094072	63	.015873016	.125988
112	.008928571	.094491	62	.016129032	.127000
111	.009009009	.094916	61	.016393443	.128037

TABLE 2 *(Continued)*—GIVING VALUES OF s AND $\sqrt{s}$.

s=sine of slope=fall of water surface (h) in any distance (l), divided by that distance=$\frac{h}{l}$.

Slope 1 in	s=sine of slope.	$\sqrt{s}$.	Slope 1 in	s=sine of slope.	$\sqrt{s}$.
60	.016666667	.129100	31	.032258065	.179605
59	.016949153	.130189	30	.033333333	.182574
58	.017241379	.131305	29	.034452759	.185695
57	.017543860	.132453	28	.035714286	.188982
56	.017850143	.133630	27	.037037037	.192450
55	.018181818	.134839	26	.038461538	.196116
54	.018518519	.136085	25	.040000000	.200000
53	.018867925	.137361	24	.041666667	.204124
52	.019230769	.138676	23	.043478261	.208514
51	.019607843	.140028	22	.045454545	.213200
50	.020000000	.141421	21	.047619048	.218218
49	.020408163	.142857	20	.050000000	.223607
48	.020833333	.144337	19	.052631579	.229416
47	.021276600	.145865	18	.055555555	.235702
46	.021739130	.147444	17	.058823529	.242536
45	.022222222	.149071	16	.062500000	.250000
44	.022727273	.150756	15	.066666667	.258199
43	.023255814	.152499	14	.071428571	.267261
42	.023809524	.154303	13	.076923077	.277350
41	.024390244	.156174	12	.083333333	.288675
40	.025000000	.158114	11	.090909090	.301511
39	.025641026	.160125	10	.100000000	.316228
38	.026315789	.162221	9	.111111111	.333333
37	.027027027	.164399	8	.125000000	.353553
36	.027777778	.166667	7	.142857143	.377978
35	.028571429	.169031	6	.166666660	.408248
34	.029411765	.171499	5	.200000000	.447214
33	.030303030	.174077	4	.250000000	.500000
32	.031250000	.176777			

ON TIDES AND TIDAL SCOUR.*

By Mr. JOSEPH BOULT, C. E.

From "The Engineer."

THAT the force producing tides is different from that observed in tidal currents is very obvious when the velocity of the tidal force in different places is compared with that of the currents. As in most places contiguous to tidal water there are, speaking broadly, tides twice in every twenty-four hours, the tidal force must run over the circumference of the globe in that period—that is, on the equator—approximately at the rate of 1,000 miles in the hour. From observations made by Mr. Rendel, in the Mersey, it appears the tidal force moves at varying velocities in that river, and that the velocity vary with the state of the tide between springs and neaps. Between Formby Point and New Brighton the rate of the so-called head of the wave appears to be uniformly 24 miles in the hour; between New Brighton and Prince's Dock, 12 miles at springs, and 6.22 miles at neaps; from Prince's Dock to Ellesmere Port the rates are respec-

*A paper read before the Liverpool Engineering Society.

tively 27 miles and 15.43 miles; from Ellesmere Port to Runcorn 28 miles and 26.67 miles; from Runcorn to Fidler's Ferry, 12.5 miles and 6 miles; from Fidler's Ferry to Warrington, 7.5 miles and 6.76 miles; the average rates from Formby Point to Warrington, 16.7 miles and 11.4 miles. As is well known, the greatest velocity of tidal streams in the Mersey does not exceed 7 miles in the hour, the average being much less. As the number of observations taken by Mr. Rendel's directions were extremely limited, the results quoted are open to correction, but for the present purpose they suffice.

On referring to other rivers, it will be seen that the differences in the rate of tidal force and tidal current are very great. In the Thames they appear as follows:

	Head of wave S.	Head of wave N.
Between Deptford and London Bridge	15.6	15.6
Between London Bridge and Battersea	9.73	14.6
Between St. Katherine's Docks and Battersea Bridge	9.	8.28
Between Battersea Bridge and Putney Bridge	18.	6.
Between Putney Bridge and Kew Bridge	22.5	17 nearly
Between Kew Bridge and Teddington Lock	7.6	4.6
The average between Deptford and Battersea	11.7	15.
The average between Battersea Bridge and Teddington Lock.	10.7	7.4

The rate of the tidal current between London Bridge and Putney Bridge at flood appears to be two miles per hour.

In the Tyne the velocities of the head of the wave are reported as follows:

	Head of wave S.	Head of wave N.
Between Tynemouth Haven and Prior's Stone	2	2
Between Prior's Stone and Ballast Office	2.95	2.95
Between Ballast Office and Howdon	10	5
Between Howdon and Bill Point	9+	7.4
Between Bill Point and Old Quay	9—	11.8
Between Old Quay and Elswick.	9.74	—
Between Elswick and Stella	—	—
Between Stella and Newbern	5.6	5.6
Between Tynemouth and Newburn	9—	9—

In the Clyde.

Between Port Glasgow and Bowling	12.28
Between Bowling and Clyde Bank	6.63
Between Clyde Bank and Glasgow	16.37
Between Port Glasgow and Glasgow	10.67

The velocities of the flood and ebb streams respectively, as—

Dumbarton Castle to Newshot Isle	0 91 F	1.6
Newshot Isle and Glasgow	0.43	0.9

In the Severn.

The greatest velocity of the tidal force	74 miles per hour.
At the same place the tidal current	4.43 "
The least velocity of tidal force	6 "
The tidal current at the same place	not given.
The greatest velocity of the tidal current	6 "
The tidal force at same place.	10 "

These examples suffice to show that the force by which tides are produced is very different in degree from that which produces tidal currents, it being remembered both forces vary with the state of the tide. On examining the time at which there is high-water F and C at different places from which reliable observations are recorded, it will be found that, with some remarkable variations, the gradual progress of the tidal force is from east to west. Apparently from this circumstance, combined with the unbroken mass of water in the Southern Ocean, between the latitudes of 30 deg. and 70 deg., it has been assumed tides are produced by an immense wave originating in that part of the world, much as if the ocean were pulled up by the combined influence of the sun and moon over an enormous area, and then suddenly or rapidly dropped again. Obviously this illustration is very misleading, for, if the ocean were so raised, and suddenly dropped, the wave would move concentrically in ever widening rings of diminishing altitude, as is seen when a stone is dropped into a pond, and the general advance in a westerly direction only would be impossible. The equilibrium theory of the genesis of tides which has hitherto been accepted as correct has been pronounced by such au-

thorities as Professors Whewell and Sir William Thomson to be inadequate for explaining various phenomena, but I am not aware that any modification of that theory or any other hypothesis has been generally adopted. A globe insulated from all external influence is assumed to acquire its form through three balanced forces, namely, the cohesion of its particles amongst themselves, centripetal force or gravitation to the center of the globe, and centrifugal force, and it is further assumed if the equilibrium of these forces be disturbed, the shape of the globe will be modified.

Supposing a body like the moon to be so near as to exercise an influence analogous to that of gravitation to the earth's center, the particles of the globe on the side next to the moon will be attracted with a greater, those on the further side with a less force than those which are intermediate. Consequently, the gravitation to the center of the particles nearest to the moon is diminished, and, therefore, if at liberty to move amongst themselves, will rise above the general level, and the more distant particles will be heaped up on the side which is turned away from the moon. Hence if the globe were at rest, the free particles would take the form of an oblong spheroid with its longer axis passing through the moon; and it may be shown from theory that the spheroid would be an equilibrium if the longer semi-axis exceeded the shorter by about 58in. In applying this illustration to the tides it is assumed that, in consequence of the rapid rotation of the earth, its spheroid of equilibrium is never fully formed, for the vortex of the spheroid has shifted its position on the earth's surface, and an immensely broad and very flat wave is formed, and follows—or precedes—the motion of the moon at some interval of time. In the open sea that interval is usually from two to three hours after the moon's transit over the meridian, either above or below the horizon.

It appears to me this theory is capable of a very important amendment in its conception. The movement of the particles, consequent upon the moon's influence, appears to me due to the greater freedom given to centrifugal force, in consequence of the earth's gravitation being to some small extent neutralized by the moon's influence; and that such centrifugal force does not cause "an immensely broad and very flat wave" following "the motion of the moon," but imparts to all the free or fluid particles on the earth's surface a tendency to fly off in a direction opposite that of the earth's rotation, just as mud flies backward off a carriage wheel; this centrifugal tendency operating on every meridian in turn, its effects being visible at some interval after the moon's transit above or below the horizon.

Allowing for the fact that the sun's influence is much less than the moon's, the phenomena of solar tides are identical, but they are not perceptible apart from the lunar tides. When the two sets of tides coincide in time and place, which they do when the sun and moon are in opposition or conjunction, the result appears in what are known as spring tides; when the sun and moon occupy intermediate positions, and are neither in conjunction nor in opposition, then the solar high water coincides with the lunar low water, or the reverse, and the result appears in the neap tides. Other changes in the relation of sun and moon are indicated in the varying levels of the tides, the result or what is termed their priming and lagging, as the sun is westward or eastward of the moon. Obviously these variations of the tide depend upon the extent to which the earth's gravitation is modified by the influence of the moon and sun, and the extent to which the earth's centrifugal force is freed from the restraint imposed by that gravitation through which the original theoretical equilibrium is constantly changed.

When the moon is on the equator, at the time of her transit, the action of tidal force may be assumed as parallel to the meridian; as the moon passes into north or south declination, her influence will be greater on one side of the equator than on the other, varying with her distance north or south; as the extreme range of declination is an area of nearly 60 deg., representing, from position, one-third of the earth's periphery, her influence in the region of this extreme northern declination will be very much greater than at the same distance south of the equator; and two results may follow: (1) The amount of centrifugal force liberated will be greater in the northern

than in the southern hemisphere, and consequently the tidal elevation be greater; (2) that part of the force which is in the northern hemisphere may be somewhat in advance of the southern part, and consequently the line of motion will not be parallel to the meridian, but more or less oblique, according to the amount of declination; from which conditions two tendencies arise: (1) That the motion of the tidal force will be conoidal, that is, revolving round the part which moves more slowly; (2) that the water raised to the greater altitude will flow downwards. Thus the height and progress of the force may be subjected to irregularities, irrespective of the local and temporary incidents to which attention will now be directed.

Though gravity is said to be a force which is transmitted, not during any interval of time, but instantaneously, its velocity, according to Laplace, being, if not infinite, at least fifty millions of times greater than that of light, yet the greatest and least tides do not happen exactly at the time of new and full moon, but at least two or three days after, even in places directly exposed to the ocean. These variations are no doubt to be ascribed in part to the irregular form and depth of the ocean, the inertia of the water, friction, atmospheric pressure and other causes, but principally to the time which elapses before the centrifugal force acquires its greatest strength and momentum. The mud thrown off a carriage wheel does not quit the tire at its highest level vertically over the axle, otherwise the mud would fly upwards; its backward course shows clearly that it leaves the tire at some measurable distance behind the vertical radius of the wheel. Similarly, the water which has a tendency to leave the earth does not rise to the zenith, but is thrown backward, that is westward of the meridian on which that tendency is acquired. Consequently those particles of water which are nearest to the moon, that is, are on the meridian she is actually crossing, have a tendency to fly backwards over the more westernly water in a film of thickness varying with the centrifugal energy, thin near the moon, and thickening to the utmost limit of the earth's hemisphere nearest to her—that is, at the distance of say 45 deg. from the meridian on which she is for the moment. Thus, the original equilibrium of three forces would be destroyed, and one of four forces take its place; in compliance with which the water in the hemisphere furthest from the moon would also be propelled eastward of her meridian, and form another cycloidal excrescence balancing that first formed.

If tidal action is referable to centrifugal force, in every body of water, however small there will be a tendency to that action; but probably, perceptible tides require for their genesis a considerable bulk of water. It may be that a minimum expanse must be combined with a minimum depth; probably from recorded observations, some indefinite bulk is necessary, irrespective of the ratio of expanse and depth; with a considerable surface extending east and west. In the Mediterranean, for example, it is not at all likely a wave originated in the Southern Ocean would give any motion to the water. Of the tides at Toulon it has been observed that the coincidence of phase of the main lunar and solar semi-diurnal tides, happening some four or five hours after the time of new and full moon, would point to the conclusion they were wholly generated in the Mediterranean, and were scarcely, if at all, influenced by any action from the North Atlantic, through the Straits of Gibraltar, the amount of retardation of coincidence of phase for those components, amounting on the western coast of Europe to between thirty and forty hours.

The great depth of this sea, in some places exceeding 1,000 fathoms, combined with its great length from east to west, makes its mass very considerable. Its seems not improbable as much centrifugal force may be liberated as will produce the effects observed: the absence of tides in the Levant is consistent with such an origin.

From the assumption that tides are produced by centrifugal action, it is apparent that the quantity of water removed by the tides is comparatively trivial. In the Amazon, sometimes called the Mediterranean of America, the basin is estimated to contain, exclusive of the Para and Tocantins, an area of 2,330,000, English miles; that is, more than one-third of South America, and equal to

two-thirds of Europe. The average discharge is 750,000 cubic feet, or more than 4,500,000 gallons per second. Wallace states that with the tide the water rises; but during the flood, as well as the ebb, the current is moving rapidly down. This takes place, he says, at the very mouth, for at the island of Mexiana, exposed to the open sea, the water is always fresh, and is used for drinking all the year round; though it appears from Bates that the lower courses, as well as the channels and bays of the Delta—that is, for 150 miles from the sea, have no proper downward current, but ebb and flow with the tide. The apparent discrepancy willl disappear if it be assumed that the flood does not carry any sea-water with it, does not effect any translation of water, the land water forcing its way out though unable to neutralize the tidal action. Being on the equator, the centrifugal force is very great. Some idea of its strength is suggested by the distance up the Amazon to which it penetrates. It was observed by Wallace in the Tapagóz, a branch of the great river. In that river, at the end of the dry season, there is but a small quantity of water, and the current is very sluggish. The Amazon, however, rises very considerably with the tides, and its waters become higher than those of the Tapagóz, therefore they enter that river and force it back; the Amazon itself is then seen to be flowing rapidly down, while the Tapagóz is flowing up. The tide rises in the Amazon considerably above Santarem, but the water never flows up; the surface merely rises and falls. Bates observed the water to rise daily with the tide 2 in. or 3 in. in a small creek of the Cupari branch, 530 miles from the sea; therefore the current in the creek was not strong enough to neutralize the tidal force, as it was neutralized near the sea.

At Para, springs rise as much as 11 ft., yet a tidal current is not perceptible. It is said there are no fewer than seven tides in the Amazon, within a length of 600 miles. The Gulf Stream has a course of 3,000 miles, 60 miles wide and 100 fathoms deep, with a velocity varying from five miles an hour to only 10 miles a day, unbroken by tidal ebb or flow.

There is a large space of still water in the Irish Sea, between Carlingford and the Isle of Man, where occurs the phenomenon of water rising and falling without any perceptible stream. This space of still water is marked by a bottom of blue mud, the surface probably of a deposit of blue clay, an unfailing indication of the absence of disturbance, since probably it is largely impregnated with vegetable matter, from which its color is derived, and characteristic of riparian deposits. The stream by which the mud is conveyed to this spot must, therefore, be very gentle.

The maximum elevation of a lunar tide is estimated to be 5 ft., that of a solar tide 2 ft. The sum of those figures represents the highest springs, and their difference the lowest neaps. The elevation of 7 ft. above the surface of the ocean is but the infinitesimal part of the earth's diameter of nearly 8,000 miles, and has a much smaller ratio to the moon's distance of 237,000 miles, or thirty diameters. The ratio of the centrifugal force, released by the influence of the moon and sun, to the earth's centripetal force, must also be very small, and to produce the effects assigned to it may be conceived as acting hydrodynamically like an hydraulic ram; that is, the centrifugal force is, as it were, in a closed vessel full of water, the top of which is, in a degree, elastic. When the force is too weak to expand the cover is confined, and under pressure; as the force is increased the cover is expanded, but when the expansion reaches the limit of elasticity the water is completely bound, and great force may be transmitted through its agency. For the elastic cover or web, acting on the surface of the water, substitute elastic gravitation acting upon every particle of water, it is obvious the effect is similar, and pressure applied at one end of the vessel of water will rapidly produce effect at the other end, whatever the distance, if the channel be unobstructed.

This conception of the resistance offered by centripetal to centrifugal force appears identical with that of "the practical rigidity conferred by rotation" on frictionless particles, which has been proposed, almost simultaneously, by General Barnard and Sir William Thomson. But they assume "an infinitely rigid envelope," instead of one partially elastic, which appears to me to represent

the action of gravitation; the phrase "*indefinitely* rigid envelope" will appear to be unexceptionable.

Continents and islands, some of varied outline, occasion interruption and deviation in the course of centrifugal force. It is not likely any appreciable force is released from land, because of the cohesion of its particles, and its density is very much more than that of water with, say, twice the specific gravity. Land surface, therefore, does not present the undulating motion so characteristic of tidal water. So, likewise, it is to be observed that, on the western side of land, the inshore water will not be raised by the passage of centrifugal force from the eastward; and the tendency of the force generated westward of the land is to lower the surface of the inshore water, through the increased energy temporarily given to centripetal force, thus producing low water; half-tide level marking the brief interval when the centrifugal and centripetal forces balance each other. This is, theoretically, the normal level of the sea when undisturbed by the influence of the moon and sun. Thus the phenomenon of contemporaneous high and low water, on the same meridian but in different latitudes, is observable in the ocean, as on the West Coast of Africa, when the level of the inshore water is reduced, there may be high water to the southward.

Another feature in the action of centrifugal force must not be overlooked, which is its greater development in equatorial than in polar regions. At the poles themselves there is not any rotation, and consequently there is not any centrifugal force: but its genesis has increasing energy till the greatest is acquired on the equator or between the tropics; thence the excess may pass north and south in the lines of least resistance, giving motion to circum-polar waters, losing therein its centrifugal character, becoming simply hydrodynamical, and, as such, returning towards the tropics, and giving to inshore waters that tidal influence which could not be bestowed by centrifugal force in its normal course.

These suggestions receive confirmation from the tidal observations made in the Arctic expedition, 1875-6. On these Professor Haughton remarks that the expedition, proceeding northwards up Smith's Sound, met the tide coming from the north, at or near Cape Frazer, lat. 79 deg. 40 min., and left the tides of Baffin's Bay. The new tidal wave, observed on both ships, is specifically distinct from the Baffin's Bay tide, and from the tide which enters the Arctic Ocean through Behring Straits; and it is, without question, a tide which has passed from the Atlantic Ocean round Greenland northward, and then westward. The Artic Ocean being for the most part covered with thick ice, may be regarded as an accumulating chamber, into which the tidal forces from the North Pacific and North Atlantic are combined under pressure, and issue southwards to produce the tides of North-Western Europe. It would be inappropriate here to enter more at length into the discussion of the general question of the genesis of tides; those who wish to do so are referred to a paper on the subject in the *British Architect and Northern Engineer* for August 1st and 8th, 1879; but preparatory to applying the views advocated to the special purpose of this paper, it is desirable to resume so much of the discussion as relates to the North Atlantic.

From the preceding suggestions it would be expected that in the North Atlantic tides of the same age on the western shores of Europe would follow those on the eastern shores of America; that is, that the time of H. W., F. and C., would be later in Europe, because the easternmost part of the Atlantic is inshore. This anticipation is confirmed by Whewell, in his remarks upon simultaneous observations, and from the tables published by the Admiralty annually. First of all, it may be mentioned that on the coast of America the tide advances from the equator in a northerly and westernly direction, resulting from the resistance presented by that continent and the islands of America to the centrifugal force. That being most developed in equatorial regions is deflected off the coast of South America northwards, and, in combination with the various degrees of force generated in each latitude, advances westward, touching first at Cape Hatteras, and thence traveling southward to Cape Fear, Charlestown, Savannah, and St. Augustine;

northward to Delaware, New York, and into the bays of Massachusetts, Boston and Fundy. At Newport the tide-hour is about midway between that of Cape Hatteras and those of Delaware and New York, its meridian being more eastern. The lateness of the hour northward of Newport is to be ascribed to the irregularities of the coast line, for headlands and islands obstruct the progress of the tidal force, as has been illustrated by the delay in that part of the force which, splitting off at the south point of Greenland, travels along the eastern coast into the Arctic Ocean, forming "the new tidal wave" referred to by Professor Haughton as observed by the Arctic expedition.

Those irregularties are, however, very trivial compared with the marked differences between the tidal hours in North America and in Europe; in the latter "the moon's crossing the equator is not felt in its effects until two or three days afterwards." Dr. Whewell further observes (27), "The different epoch of the diurnal inequality in different parts of the world is a very curious fact; the more so, since it is inconsistent with the mode hitherto adopted of explaining the circumstances of the tides, by conceiving a tide-wave to travel along all the shores in succession. In accordance with that view the tides on the shore of America had been considered as identical with the tide on the coasts of Spain and Portugal, which occurs about the same moment; nor does it appear easy to imagine the form of the tidal-waves so that this shall not be the case. Yet we find that the tides on these two sides of the Atlantic cannot be identical in all respects, for on 9th, 10th, and 11th June, when the diurnal inequality was great in America, it was nothing in the west of Europe; and on the 18th and 19th, when this inequality had vanished in America, it was great in Europe. It would seem as if the tidal phenomena on this side of the Atlantic corresponded to an epoch two or three days later than the same phenomena in America; and we may, perhaps, add, that different kinds of phenomena do not appear to travel at the same rate. Thus the equilibrium theory, though it may explain the general form of the inequalities, cannot give their epochs and amounts by any possible adjustment of constants. I may add, he says, that the notion of the progress of the tide-wave from south to north in the Atlantic is still further involved in difficulties by its appearing at the Cape of Good Hope, that the diurnal inequality showed itself most clearly on 17th, 18th, and 19th June, that is as late as in Spain and Portugal."

Just as the lateness of the tides on the western shores of Europe appears due to the time occupied by the return of the tidal force from the east of North America, after its northern deflection, so the similar lateness on the west coast of South Africa may be due to the return of the motor from the east coast of South America. There is much complexity in the tidal action off the west coast of South America, arising probably from the combined influence of the two deflections, caused by the east coast of Africa and the east coast of South America; and uncertainty as the distribution of land and water in antarctic regions precludes the possibility of a satisfactory explanation.

On referring to the tidal hours of the British Isles, it will be found that the earliest point of contact is the islet of Rockhall, long. 13 to 14 deg. W., lat. 58 deg. N., where H.W., F. and C. is at 3.30. On the west coast of Ireland the earliest time is also 3.30; that is at the Blaskets, off the extreme point of Kerry, long. 12 deg. At the extreme west point of France, the Isle of Ushant, the hour is practically the same, or 3.32; at Cape Ortegal and Finisterre, the north-west extremity of Spain, the time is 3.0; and at Belem, Lisbon, 2.30, and the hour gradually becomes earlier—or later—to the Straits of Gibraltar, and the West Coast of Africa, suggestive of a meeting in the Bay of Biscay between the North Atlantic return and an offshoot from the equatorial regions of the African Coast. Rockhall at the Blaskets are nearly on the same meridian. From the latter the tide hour gradually advances eastward, not westward, reaching Carnsore Point, on the south, at 6.0, Tuscar at 5.45, and Ballycastle Bay, on the north, at 6.25. From Carnsore, the hour advances rapidly to 11.0 at Carlingford Point. On the north the tidal force is detained in the narrow passage between Fairhead and the Mull of Cantire, and does not pass Red Bay until 10.31, reaching Don-

aghadee at 11.13. On approaching Scotland the tide hour at St. Kilda is 5.30, at Bernera in the Western Isles 6.11; its further progress eastward is very irregular and slow until it passes the Mull of Cantire at 10.45, reaching Morecambe Bay at 11.26; the numerous islands and litoral irregularities of the west coast of Scotland cause much delay. At the extreme north of the Shetland Isles, where the force is divided, the tide hour is 9.45, thence it advances southward and eastward, with considerable regularity of increase, to the mouth of the Thames, the hour being very uniform at places in the latitude of the Moray Frith and Kinnaird's Head; and just twelve hours later at the Thames.

On the west coast of England the tide hour from the south advances from 4.30 at the Scilly Isles, until it meets that from the north around the Isle of Man. Passing eastward through the English Channel, the hour advances steadily to Portland Bill, a little eastward of which it is detained for two or three hours by the contraction arising from the projection of Cape de la Hogue; afterwards it advances slowly and gradually through the Straits of Dover to the mouth of the Thames.

On the coast of Norway the tide hour at the Romdals Islands, a few degrees north of the Shetlands, and about 6 deg. east, the tide hour is 10.45, or just an hour later than at the north of the Shetlands. At the Loffoden Isles, considerably north and east of the Romdals, the hour is noon. Going southwards there is a detention between the Shetlands and Norway, and the hour at Bergen is 1.30, about the same as it is at Arbroath in Scotland, showing a drag, caused doubtless by the rugged coast of Norway and the in-draught to the Baltic. At the Skawt, south of the entrance to the Baltic, the hour is 5·56; here the hour from the north meets that which traveled through the English Channel along the coasts of France, Belgium, Holland, Germany and Denmark. In thus tracing the progress of the several divisions of the tidal force in these parts of Western Europe observation has been confined to the most prominent features of the respective coasts, such as headlands or islands, so far as they can be obtained from the Admiralty tables. The detention caused by islands has been referred to in the case of Greenland and the isles of Scotland; there are two other instances, on the coasts of England and Ireland, which are worth special notice. In the Solent, and as far to the westward as Portland, there are what are termed the first and second high waters. After low-water the tide at Southampton rises pretty steadily for seven hours, which may be considered as the first or proper high-water; then for an hour it ebbs 9 in., at the end of which time it begins again to rise, and in about 1¼ hours reaches its former level, and sometimes higher; this is called the second high-water. The tidal level is therefore nearly stationary for rather more than two hours; similar first and second high-waters occur on either shore of the Solent. This phenomenon is ascribable to the tidal force being divided at the Needles, one part traveling up the Solent, passing Hurst Castle at ten o'clock and West Cowes at 10.45, the other passing to the southward, and turning round Bembridge Point at eleven o'clock into Spithead, reaching West Cowes at 11.45 and Hurst Castle at noon. At Havre the water remains stationary for an hour, with a rise and fall of 3 in. or 4 in. for another hour, and it rises and falls 13 in. only for the space of three hours. This irregularity, no doubt, arises from the sudden projection of Cape de la Hogue, combined with the peculiar formation of the coast of Havre. On the coast of Wicklow County, and abreast of the Arklow Bank, is Courtown, a place which is termed a node or hinge of the tide, where it is often said the tide neither rises nor falls. This spot appears to me another example of the division of tidal forces here by the Bank of Arklow. Courtown is about midway between Wexford Harbor and Wicklow, and protected by the bank from any direct approach from the sea. There is a difference of four hours in the tidal establishments of Wexford and Wicklow; that is, high-water is four hours earlier at Wexford Haven than at Wicklow Head. At Kilmichael Point, a little north of Courtown, the establishment is exactly two hours later than at Wexford Haven, and two hours earlier than at Wicklow Head. The tidal range is 2 ft. at Wexford, and Wicklow six or seven

feet. It is clear the times of high-water at one end of the channel of Courtown and of low-water at the other so nearly coincide, the tides become complementary, and nearly balance each other at Courtown, producing a state of almost no-tide. To a limited extent the phenomenon of two tides is found in the Mersey, the flood through the Horse or Hoose and Rock channels, being twenty minutes to half an hour in advance of that through the main channels, which is detained by Great Burbo and other banks. On the ebb the tide, I think, usually turns earlier on the Cheshire than on the Lancashire shore, but not always; in making observation it is necessary to be very careful, as light bodies on the surface may drift down the river while heavier bodies are still carried upwards.

From observations reported by Mr. Shoolbred, C. E., it appears that the time of high-water of an equinoctial spring tide, April 8th, 1875, is nearly uniform at Whitehaven, Fleetwood, Liverpool, Belfast, Dundalk, Dublin and Kingstown, the greatest difference between any two places not exceeding forty-five minutes, whilst it is the same in four places out of seven, two of the other three being identical. At Barrow and Holyhead the variations were considerable, being much earlier at Holyhead and much later at Barrow. All these places border the area round the Isle of Man, in which the forces from the north and south unite. Admiral Beechey observed that the time of low-water at the northern and southern entrances of the Irish Sea, and at the entrance of the English Channel, were identical with that of high-water in Morecambe Bay and the Straits of Dover, and the reverse. Taking Mr. Shoolbred's observations, for the convenient comparison of the tidal range on the west coast of England and the east coast of Ireland, it appears that at—

	Feet.
Whitehaven the extreme range is...	27
Barrow " " " ...	30.66
Fleetwood " " " ...	28.93
Liverpool " " " ...	29.75
Holyhead " " " ...	18.5
	5)134.83
Mean.....	26.96

and at

Belfast " " " ...	10.0
Dundalk " " " ...	14.33
Dublin " " " ...	13
Kingstown " " " ...	12.5
	4) 49.83
	12.46

That is, the range in England is twice as great as it is at the other side of the Channel in Ireland; and, where the times of high-water on each side coincide, the surface is inclined from east to west; at low-water the inclination is reversed, or, as it may be briefly expressed, at high-water the current is from England and Wales to Ireland; at low-water from Ireland to England and Wales. The explanation of this phenomenon is to be found probably in two facts, viz., the reunion of tidal force in the area round the Isle of Man and in Morecambe Bay, and the much greater volume and velocity of the streams in England and Wales as compared with those in Ireland. If a person carrying water in an open vessel stop suddenly the surface of the water will rise higher, if he has been moving quickly, than with a slow movement; and so, though in the ocean the surface of the water is not usually much elevated, yet, if any obstruction present itself, the rebound of the force causes an increased elevation according to the size and abruptness of the obstruction. In meeting a land stream the result is similar, with this additional feature, that the surface of that stream is also raised, action and reaction being equal; and when the obstruction is removed by the advance of the tidal force a reaction follows which carries the surface of the water about as much below its mean level as it had been raised above, unless the basin is too shallow. The ebb, then, is not caused by any continued tidal influence, but is the return of the water which had been dammed back by the tidal force during its passage up the river, in which it finally expires at the highest attainable limit.

The motion of this force may be conceived of as resembling the undulation of a carpet or table-cloth when a little air has been caught between it and the floor or table, but the particles of the carpet or cloth do not flow down each side of the undulation as do the particles of water. In the paper before mentioned I

have compared the action of tidal force in channels of irregular section to the rapid propulsion of an elastic dam, which contracts and expands with the form and size of the channel. Assuming the vertical section to be parabolic, the dam forms a weir of two slopes, up which at each stage of advance the upland waters rise on the one side to descend upon the other; there being two streams on the upward face of the dam. one over the other, the depth of each being undefined, analogous to the contrary movements in bodies of different draught referred to above. Other things being equal, the height of the weir will vary with the depth of the channel, because the level of the crown of the weir, that is, of high water of any tide, is uniform, or nearly uniform, throughout the tidal portion of a river. Thus the reason why dredging has been so serviceable becomes apparent, not, as is usually said, because more tidal water passes into the river, for, if the views enunciated above are correct, tidal force is as viewless as the wind and as immaterial, but because more upland water is pent up and afterwards discharged, as when sewers are flushed by a shallow stream. All attempts to force or to coax more sea-water into a river have almost always, if not uniformly failed; where said to be successful, the increased elevation of the level of high-water has been very trivial. On the other hand, where the altitude of the tidal weir has been increased by lowering its base, that is by dredging, the effective force of the upland waters has been increased and the channel deepened. This result is conspicuous in the Tyne, the Clyde, the Tay, the Ribble, the Liffey, and other tidal harbors.

I have been favored by Mr. Stoney, of Dublin, with interesting information respecting the improvements in the port of Dublin. The history of the operations in Dublin Bay has been recorded by Mr. J. P. Griffith in a communication to the Institution of Civil Engineers, May, 1879. The bar is from five to six miles east of Carlisle Bridge; in 1819 there were 6½ ft. of water on the bar at low water; in 1822, 8½ ft.; in 1838, 10½ ft.; in 1856 13 ft.; in 1868, 15 ft.; and in 1873, the date of the last Admiralty survey, 16 ft. There does not appear to have been any subsequent survey.

The first attempt at improvement was the adoption of dredging by hand, if that can be called dredging, and the continuation of the south bank of the Liffey to Poolbeg lighthouse, just within the bar, combined with the construction of the great north wall from the shore at Clontarf to about 1,000 ft. short of the Poolbeg light, thus leaving an entrance of that width. This work was completed by 1825, and in 1838 the depth of water on the bar had been increased to 10½ ft., or by 2 ft. since 1822, giving an average of 1½ in. per annum. Between 1838 and 1873 the depth was further increased to 16 ft., giving an average annual increase of 1.88 in.; the rate for the five years between 1868 and 1873 being 2.48 in., and the greatest on record. This increased rate appears to be due to the great increase in the amount of dredging. Mr. Griffith appears to ascribe the deepening of the bar almost exclusively to the great northern wall, the effect of which would seem to be merely that of contracting the sluice through which the upland waters are discharged, after the water level has fallen below half-tide level, that being the height of the southern part of the wall for a length of 500 ft. It is obvious that the water which enters the harbor over that part of the wall, and through the gap at Sutton, will return by the same route; and that the most obvious way now available for increasing the scour over the bar is to increase the capacity of the harbor below half-tide level, or to raise the lower part of the northern wall.

Messrs. Giles and Halpin in their report of 6th May, 1819, explain their reasons for recommending the construction of the great north wall to be the sheltering of the harbor, the prevention of the sand passing from the North Bull into the harbor, and the admission of as large a body as possible of tide-water into the harbor, and its return past the lighthouse within such limits and in such direction as will produce the best scouring power to deepen the bar, combined with the least obstruction to the navigation. They proposed that the wall should extend to the distance of 2,000 yards south-east from the shore of Clontarf, and there to commence a parabolic curve towards a point due north of the lighthouse, by which curve the direction of

the ebb current would be graduated to a suitable course for effecting the best scour at the entrance and on the bar; and the proper width of the entrance will be practically determined as the embankment proceeds by its operation upon the currents in the channel.

When Mr. Telford made his report of 16th August, 1822, the wall had been carried for a distance of about 5500ft. to its full height, about 1500ft. further to the level of H. W. N. T., and about 500ft. more to the level of half flood. The effects of the wall were very evident, for the sandbanks inside, having been deprived of their usual supply from outside, had been lowered on the average more than 3ft.; that part of the bar immediately opposite to the harbor mouth had likewise been lowered from 6½ft. to 8ft., and even 9ft.; the western part of the bar, adjacent to the old channel, remaining 7ft. and 7½ft. only. Telford was of the opinion that the entrance would still be considerably too wide to produce the full effect of deepening the bar, or forming a perfect and direct channel within the south wall; and, therefore, strongly recommended the adoption of Mr. Halpin's proposal to add a still further extension of about 500ft., to be carried forward with great caution, observing the effects produced upon the current of the flowing and ebbing tides during the ensuing winter; the extension, at present, not to be raised above the level of half tide until the effects on the current have been fully and carefully ascertained, after which the most advisable measures for completing this great work, both as to shape, extent, and height, may with propriety be determined. The mean annual discharge of water is stated to be 51,000 cubic feet per minute; the limit of tidal range up the river, a weir at Island Bridge, three and a half miles above Ringsend, or eight miles above Poolbeg; the greatest velocity of the tidal current over the bar at half ebb H. W. S. T. three and a half miles per hour; but in all tides the velocity diminishes considerably as the stream spreads and enters the deep water east of the bar. The depth of water on the bar H. W. O. S. is 28ft. Mr. Griffith, in his reply to my inquiry made before seeing Mr. Mann's book, informs me that the mean discharge of the Liffey and Dodder combined may be taken as 40,000 cubic feet per minute; and adds, no accurate gaugings have ever been taken, but the foregoing estimate, he believes, is near the truth. It is very desirable that the mean discharge of all important rivers be accurately ascertained, not only as a basis of comparison with other rivers, but also for determining the utmost amount of scour which can be made available; the amount actually made use of being as much as is required for the purpose. Before proceeding to review other harbor works, attention should be directed to the excellent method adopted by Mr. Storey for constructing the basement of retaining walls below low-water level without the aid of coffer dams, pumping, and staging. Blocks of masonry 29ft. high, 11½ft. long, and 21ft. 4in. broad at the base, are built on a wharf, and about three months after completion are lifted by powerful shears and conveyed to their destination, where each forms 4½ft. in length of the lower portion of the wall as high as low-water level. When a number of blocks has been thus laid in position, the superstructure is carried up to the coping level by tidal work in the usual manner; the total height of the wall being 45ft. Each block weighs 350 tons.

In the Clyde the improvements have been specially directed towards widening and deepening the harbor of Glasgow. The general condition of the river appears to have received very little attention. Practically, the channel from Greenock to Glasgow is now level, with the same rise of tide at each end, about 24 miles apart. The works carried out since 1758 have lowered the level of low water 8ft. at the rate of .84in. per annum; since 1853 the depression has been fully 18in., at the rate of 1in. per annum; the level of high water in 1872 was about 10in. higher than in 1853. The river has been very much widened at Glasgow. A few years since the weir, which, at 120 yards above Albert Bridge, formed the eastern limit of the harbor, was removed, with the effect of lowering rather than raising the level of high water. The weir may have caused a slight local heaping up of tide by its obstruction. The bed of the river above the weir has been lowered some feet, chiefly through the action of floods, the

soil carried away having been deposited in the harbor, and thence removed by dredging. Smeaton reported that at Pointhouse Ford, now the western end of the harbor, there were depths at low water of 1ft. 3in., and at high water 3ft. 8in.; and at Hirstpoint, now within the harbor, there were depths of 1ft. 6in. at low water, and of 3ft. 3in. at high water; in 1872 there were 14ft. at low water, and 24ft. at high water at each place. Thus the bed of the river has been lowered nearly 13ft. In 1837 springs rose and fell at Port Glasgow about 11ft., and at Glasgow 7ft. At Port Glasgow they are still about the same, but at Glasgow springs now rise 10½ ft., neaps 9¼ft.; in 1837 the time of high water was 1 hour 20 minutes later at Glasgow than at Port Glasgow—now 1 hour 5 minutes. In the beginning of the century the difference was three hours. Since 1856 the city has not been inundated; in that year the water just rose above the quay. In April, 1880, and March, 1881, two floods occurred, the first said to be the greatest for fifty years, and the second heavier still. Both caused a strong current in the river, and the removal of much sand and soil from the upper reaches of the river, which were deposited in the harbor, from which they had to be removed by dredging. After the flood of 1880 it was necessary to dredge 300,000 cubic yards, at the cost of £7,500, to restore the harbor to its previous depth; the flood of this year, 1881, would probably render as much dredging necessary for the removal of the silt. During the quarter which ended about the time of the flood, 318,112 cubic yards had been dredged, of which 273,936 yards were new material, and nearly 17,000 yards were deposit. According to Mr. Ure, the ordinary rate of water discharge is 48,000 cubic feet per minute. Since the removal of the weir mentioned above, the influence of the tide has been observed 1,400 yards beyond its former range. These particulars are compiled chiefly from a communication to the Inst. C. E., "Proceedings," 1872–73, by Mr. Deas, engineer-in-chief to the Clyde Navigation. I have been favored by Mr. Deas with copies of his report on the bed and banks of the river Clyde, from the site of the weir above Albert Bridge to Westthorn, 1880, and of his quarterly reports, July, 1880, and April, 1881. I have also to acknowledge the courteous assistance of Mr. John D. Parker.

In the river Tay the mean or ordinary discharge is 274,000 cubic feet or 7,645 tons of water per minute; the river is navigable to Perth, which is twenty-two miles above Dundee, and thirty-two miles from the German Ocean. Before the commencement of the works for improving the navigation, which appear to have been confined to a length of nearly 12,000 yards below Perth, the passage was obstructed by certain ridges called fords, which stretched across at different points between Perth and Newburgh, and vessels drawing from 10ft. to 11ft. were unable, without great difficulty, to make their way from Newburgh to Perth during the highest tides. The most objectionable of these fords were about six in number, and the depth of water upon them varied from 1ft. 9in. to 2ft. 6in. at low water, and from 11ft. 9in. to 14ft. at H. W. S. T., so that the regulating navigable depth could not be reckoned at more than 11ft. under the most favorable circumstances. In addition many detached boulders were scattered over the bottom, and numerous fish cairns of stone and gravel had been constructed by the salmon fishers. The works for improving the navigation, extended over six working seasons, consisted of removing the boulders and fish cairns, and of dredging the fords, harrowing being adopted in some of the softer banks in the lower part of the river; three subsidiary channels were cut off by embankments, and in the most contracted parts the banks on both sides were excavated below low-water mark so as to equalize the currents. The depths at the shallowest places were made pretty uniform, being 5ft. at low water, and 15ft. at H. W. O. S.; steamers of small draught can now ply at low water, and vessels drawing 14ft. can get up to Perth in one tide with ease and safety. The velocity of the tidal force in this part of the river has been increased more than 1⅞ miles per hour, and the period of its influence at Perth has been extended fifty minutes.

In 1833 spring tides flowed.....	2h. 20m.
" " ebbed.....	7h. 0m.
Neap tides flowed.....	3h. 15m.
" " ebbed.....	7h. 0m.

In 1844 spring tides flowed.....	3h. 10m.
" " ebbed.....	7h. 0m.
Neap tides flowed.....	3h. 10m.
" " ebbed.....	7h. 0m.

The fall on the surface of the river from the tide basin at Perth to Newburgh, in 1833 was 4ft., and is reduced to 2ft. Between Dundee and Newburgh the river has not been improved and the tidal phenomena are unchanged. In respect to the Tay, a remarkable instance of the difficulty experienced in ascertaining facts is presented by the reports of Captain Washington, R. N., and Mr. James Walker, C. E. The former in a report to the *Hydrographer* to the Admiralty, dated 1st October, 1844, says:—"Tay bar no longer exists, and vessels drawing 24ft. of water may now enter the river and bring up in safety within its shelter at the lowest water of spring tides." Mr. Walker, writing 21st January, 1845, less than four months later, says:—"The general result of the various information I received, and my conviction from that information, is that the depth of the entrance has not varied during the last century and a-half—being 18ft."

In the Ribble the ordinary discharge of water per minute is 139,935 cubic feet.

Before the improvements were commenced a solid ridge of sandstone, about 300 yards in length, stretched right across the channel about half a mile below Preston; its surface was from 2ft. to 5ft. above the general bed of the river, and the higher parts of the rock were occasionally left dry during the long droughts of summer. At Lytham, 12 miles below Preston, the ordinary rise of spring tides is 19ft. and of neap tides 14ft.; at Preston the rise of spring tides did not exceed 6ft., and the neap tides did not reach Preston, At "the chain" about two miles below the town, the level of low water is now left 8in. lower than in 1841, before which time the works had begun to show their effects; so it may be concluded that the total lowering is between 7ft. and 8ft., and that the tidal range has been increased to the same extent. The tidal propagation has been accelerated upwards of an hour. Training walls and dredging have combined to produce these satisfactory results, through which vessels, to which the navigation was formerly closed at all times, can now get up to the quays at Preston, even at neap tides, with comparative ease and safety.

The works on the Tyne are the most important which I propose to bring under your notice; they extend as completed, from the outside of the site of the former bar to 2½ miles above Newcastle Bridge, a total length of 13 miles, and they are in progress upwards for a further length of 6¼ miles. The works were commenced in 1853 by enclosing a bight of the river about three miles from the bar, and converting it into the Northumberland Dock of 55 acres of water, exclusive of basin and lock; it was opened in October, 1857. In 1856 were commenced the piers at each side of the river entrance. They were designed by Mr. James Walker, C. E., for the protection of vessels from the prevalent and destructive N. E. to S. E gales, and for facilitating the removal of the bar. In 1859 Mr. J. F. Ure, C. E., formerly of the Clyde, prepared a comprehensive scheme for the improvement of the tidal portions of the river, extending from the bar to the boundary stone, Hedwin springs, a distance of 19¼ miles; the requisite parliamentary powers were obtained in 1861, and the following results realized:—The bar has been removed, and, where formerly there was only a depth of 6ft. at low water, there are now more than 20ft., a depth which is maintained for a considerable width into the harbor. In 1879 the depth of water at the entrance of the Tyne was not less than 22ft. at low water, or 37ft. at H. W. S. T. The Narrows, below South Shields, have been widened from 400ft. to 670ft. In Shields harbor dangerous shoals have been removed, and for a length of 8000ft. vessels can moor in a depth of more than 30ft. L. W. S. T., and instead of the tortuous channel through which it was formerly difficult to navigate one vessel at high water, three or four may now be safely towed abreast at any time of tide. Vessels which formerly were detained for months before the bar could be crossed in safety even at high water, can now leave in any weather in which it is fit to go to sea, at or within a short time of low water. For the whole distance between Shields harbor and Newcastle there is now a depth of 20ft. L. W. S. T.,

where river steamers drawing 3ft. or 4ft. water used to ground for hours. For 2½ miles above Newcastle the river has been deepened 18ft. low water, and at Blaydon a depth of 12ft. has been made. In the upper part of the river a new cut, 400ft. wide, has been made through Lemington point, and opposite Blaydon the river has been widened from 150ft. to 400ft., thus enlarging the tidal receptacle. There has not been much alteration in the tidal range, the greatest, 2½ft. is at Hebburn quay, where L. W. S., ebb upwards of 2ft. lower.

In the floods the liability to inundation has apparently been removed, as the flood line on the 10th of March last was considerably lower than in the floods of November, 1856 and 1861, though at Wylam Bridge, above the boundary stone, the highest level in 1881 was 3in. above that in 1861, and 8in. only below that of 1856. Before 1860 high water at Newcastle Bridge was sixty minutes later than at the entrance of Shields Harbor, a distance of about 9½ miles, and at Newburn, about 7½ miles above Newcastle, it was twenty-nine minutes later still. In 1879 high water at Newcastle was only twelve minutes later than at Shields, being a gain of forty-eight minutes, and the gain at Newburn is twenty-one minutes. It does not appear that the mean annual discharge has ever been ascertained. For the interesting information respecting the Tyne, except as specially mentioned, I am indebted to Mr. Philip J. Messent, the acting engineer to the Tyne Commission.

The beneficial effects of the works referred to may be presented in the works of Mr. David Stevenson, who has summarized them as follows: (1) To depress the level of low water; (2) to increase the range of tide; (3) to accelerate the propagation of the tide through the channel of the river; (4) to prolong the duration of tide in the river; (5) to equalize the velocity of the tidal currents, removing rapids and bores; (6) to add to the beneficial scouring power of the river; (7) to increase the navigable depth.

It has been shown that in the harbor of Glasgow the bed of the Clyde has been made practically level; but Mr. Stevenson, speaking from his own experience, suggests that an engineer may calculate on reducing the slope of tidal navigation to about 3in. or 4in. per mile—equal to a gradient of 1 in 15,840; and that the gradient should not, if possible, exceed 10in. per mile, which is equal to a gradient of 1 in 6,336. It is therefore most desirable that all strictures, horizontal or vertical, should, as far as possible, be removed, especially those which abruptly interfere with tidal propagation. Their removal, while prolonging the period of tidal action, will reduce the velocity of tidal currents, and so facilitate navigation and relieve it from hazardous contingencies.

That the widening or deepening of any part of a river increases its tidal capacity is sufficently obvious, but the manner in which that capacity becomes available may be worth some consideration. It is usual to say, if the receptacle be provided, the tide will throw into it a larger volume of sea-water; but that statement is based on the assumption that the tide acts as a wave, and in some situations effects the translation of a large body of water. If the suggestions offered in the first part of this paper are well founded, tidal action, so long as it lasts, is not intermittent, but continuous, whereas all wave action is essentially intermittent; in the ocean, sea, and river, usually caused by wind. No doubt the rise or fall of the tide is generally accompanied by wave motion, if only a ripple; but, as before stated, there is in the Irish Sea a spot where the water rises without any current or wave, except such as is due to local and special disturbance, and in very calm weather the tide may be observed to rise upon any shore with scarcely a ripple to break the surface of the water. Therefore, instead of assuming that the tide forces sea-water into an estuary or river, it would appear to be more correct to say the tidal force dams back the fresh water brought down the river, and causes an indefinite number of flushes, or a continuous flush, the place of discharge being momentarily removed higher and higher, until the flush ceases, because the level of the top of a weir is reached. Thus, then, the element in calculating the extent to which a receptable should be enlarged is the quantity of water discharged by the river in the period during which the tide is rising at any part of the river's course.

The more the course of the weir is

left free from obstruction arising from irregularities in the bed or shores of a river, the more rapidly will the tidal force advance up the river and the less inclined will be the plane presented by that force to the descending water, consequently the more rapidly will that water rise, and it will attain to a greater altitude, just as in a stream, the level above an obstructive boulder is greater than it is above a pebble.

Attention has been drawn to the existence of contrary streams in tidal rivers, arising from the greater specific gravity of salt water as compared with fresh. This was first observed by Mr. Robert Stevenson in 1812, in the Aberdeenshire Dee; he noticed that the current of the river continued to flow towards the sea with as much apparent velocity during flood as during ebb-tide, while the surface of the river rose and fell in a regular manner with the waters of the ocean. By the use of a hydrophore, Mr. Stevenson ascertained that the salt or tidal waters of the ocean flowed up the channel of the Dee, and also up Footdee and Torryburn, in a distinct stratum next the bottom, and under the fresh water, which continued perfectly fresh and flowing in its usual course towards the sea, the only change discoverable being in its level, which was raised by the salt water forcing its way under. The tidal water so forced up continued salt, and when the specific gravity of specimens from the bottom were tried and compared with those taken at the surface, the lower stratum was always found to possess the greater specific gravity due to salt over fresh water. Mr. Alan Stevenson, judging from marine productions, considered that salt water in the Tay rose to a very short distance above Mugdrum Island, that is, twenty-five miles from the mouth of the river and ten miles below the tidal limit.

I have not been able to obtain any information as to the thickness of the stratum of salt water, though it is impossible to form any accurate idea of the quantity of water forced in from the sea, without the measurements of length, breadth, and thickness. On the assumption that tidal force is exerted as a weir, a small quantity of sea-water would be carried by the force up the river, but in consequence of its greater specific gravity, the water so carried in would have a tendency to the bottom of the weir. There, through irregularities on the bed and sides of the river, it would be gradually mixed with the fresh water until its marine character was lost, the quantity of salt water, always a vanishing quantity, being gradually reduced from the original amount, which, probably, is very small compared with the fresh water brought down the river.

It has been shown that the effect of improving the navigation of a river is to increase the velocity of the power which produces tides, and to diminish the velocity of tidal currents; and if the currents flow with less rapidity in the same time, it is clear the quantity of water translated by tidal influence must be less, and consequently, that less water is forced into the river from the sea. Thus it follows that the back-water, on which the scour of rivers depends, is so much of the upland waters as is penned back by the weir, until it accumulates to high water, the highest attainable level, and is then discharged. In improving harbors, therefore, the aim should be so to enlarge the receptacle above the weir that it may retain all the water the river discharges during the rise of an ordinary spring tide. If the receptacle is less than will accomplish this, useful water is lost. If it is larger, the cost of the works will probably be more than the value of the water stored, and deposits of silt will be formed. I am disposed, however, to accept the conclusion of the special commission on the Tyne, appointed in 1854—"that not only should every river, as observed by Mr. Rendel, be treated according to its own peculiar character, but, strictly speaking, each part of the same river may require different management.

The comparatively small influence usually ascribed to floods upon the enduring features in the tidal part of a river confirms the opinion that there is no adequate advantage in increasing the means of retaining water beyond the level of O. S. T., because the scouring tendencies of such excessive back-water are to some extent counteracted by the recurrence of tidal opposition. To that circumstance must be ascribed the large deposit in Glasgow harbor after the great flood in March last.

It is important that the amount of back-water, should yield the requisite amount of scour before the return of the tide; any excess tends to cause streams of inconvenient velocity until they are reduced by tidal opposition, and deposit the silt they have taken up to repletion.

What is termed the tide of ebb is generally regarded as the most effective scour, and the principal agent for maintaining the accessibility of harbors. If, however, tides are caused by centrifugal force, the term ebb tide is incorrect, as the downward current of a stream is due to gravity relieved from the opposition of centrifugal force; that is, the upland waters descend not only during the ebb, usually so-called—that is, until the tide water subsides to the river's normal level—but also during that further interval until the incoming tide produces slack-water.

NOTES ON THE RELATIONS OF MANGANESE AND CARBON IN IRON AND STEEL.

By MONS. ALEXANDRE POURCEL, Terrenoîr, Loire, France.

Transactions of the American Institute of Mining Engineers (advance sheets).

The perusal of Mr. Willard P. Ward's "Notes on the Behavior of Manganese to Carbon," presented at the Washington Institute in February, 1882, has suggested further reflections on the same general topic, and has led to the preparation of the present paper.

The same observation that Mr. Ward has put on record in his "Notes" was also made by myself at about the same time (in August, 1875), and under almost the same conditions. From a blast-furnace that was *very hot*, as was the furnace mentioned by Mr. Ward, I obtained a pig-iron containing about fifteen per cent. of manganese, gray in color, and very tough. It could be pulverized, but could not be cut with a chisel. I analyzed this iron and found that it contained, as I had suspected, a large amount of silicon. From this fact I drew the conclusion that the silicon had deprived the manganese of its power of dissolving carbon, since the latter, instead of occurring in the pig in combination appeared as graphitic carbon. I thus saw reproduced on a large scale, and demonstrated in a visible way, the property that Colonel Caron, a French scientist, had discovered in silicon—the property of obstructing the process of hardening in steels by keeping the carbon in the graphitic condition.

An attentive study of the conditions under which the phenomenon observed by Mr. Ward takes place, led me to go back to operations of synthesis, and to make as I wanted them pig-irons containing varying quantities of silicon, manganese, and carbon. An iron thus prepared was intended to serve me as a chemical reagent in the production of steels cast without blow-holes, such as my lamented friend, Mr. A. L. Holley, has introduced and made known to the United States. What I needed in order to make very soft steels, cast without blow-holes, was an iron which, when it was added to the bath of steel, introduced into the bath a sufficient amount of silicon and of manganese, with the smallest possible proportion of carbon. Now, in analyzing an iron similar in character to that obtained by Mr. Ward, I found that the amount of combined carbon in the iron was almost nothing, and that the total carbon was between 3 and 3½ per cent., instead of being from 5 to 5½ per cent., as in ordinary spiegels containing 15 to 16 per cent. of manganese.

I then sought for a way of still further diminishing the carbon by increasing the silicon and manganese, and after a few trials I found that when the manganese and silicon are present in the ratio of their chemical equivalents, the carbon reaches a minimum. It is well understood that the higher the percentage of manganese and of silicon in the pig is raised, the lower the percentage of carbon will be; an almost complete elimination of carbon might, indeed, be obtained by means of silicon, but the law which determines that the percentage of carbon

shall reach its minimum is fixed by the ratio Mn : Si. When the manganese increases, the carbon increases also. For example, I have produced a number of tons of iron with from 11 to 13.5 per cent. of silicon, and from 17 to 19 per cent. of manganese, and the percentage of carbon has been the least—2 per cent., —with 13.2 per cent of silicon and 17 per cent. of maganese, that is to say, when the two substances are present in the ratio Mn : Si.*

What are the reactions that take place in the blast-furnace when a pig-iron, or rather an alloy, of this kind is produced? Are the phenomena simultaneous or successive? My opinion is that they are successive, and that the carburet of manganese is the reagent that reduces the silica from which silicon is derived. We can in fact repeat that laboratory experiment which consists in maintaining a quantity of ferromanganese in a molten condition for several hours in a thick crucible, such as is used in the melting of steel. According to the length of time, more or less, that the ferromanganese is kept in the molten state, we find the walls of the crucible to be more or less attacked; the metal incorporates with itself a notable quantity of silicon, and loses some of its manganese and carbon. In this experiment there can be no doubt that the carburet of manganese is the reagent by whose action the silicon is derived from the silica in the walls of the crucible.

The laws of thermochemistry that have been established by Berthelot's numerous fine experiments equally confirm the opinion, to which I some time ago gave utterance, that when silicon and manganese occur together in a pig-iron or in a steel, they are in a state of chemical combination, as a silicide of manganese, if the percentages of the two substances are in the ratio, at least, of Mn : Si. It may, indeed, be affirmed that silicon when neutralized by manganese, that is to say, when for each chemical equivalent of silicon there is present a little more than an equivalent of manganese, does not diminish in the least the hardening property of steels. When the amount of manganese increases, the hardening property increases, since the manganese possesses the property of dissolving carbon, that is to say, of keeping it in the combined state.

As to the opinion of Mr. Ward that maganese has no injurious effect on the wear of rails I may say that I hold the same opinion, though for an entirely different reason from that given by Mr. Ward. The deterioration of rails from atmospheric causes, which may be likened to chemical action, is due especially to their physical condition rather than to the chemical composition of the ingot, from which the rail was made. A porous ingot, full of blow-holes, will produce a rail, on which, after a few months of service, the surface exposed to wear will be covered with numberless little rays or streaks, which are just so many more points of attack for atmospheric agents. Such a rail if laid in a damp tunnel will very quickly become useless. Possibly it would be used up a little more rapidly if it contained a high percentage of manganese, but in no case would the presence of that element be a principal cause of the effect produced.

If two rails, made from two ingots perfectly sound and free from blow-holes, are compared with each other as regards mechanical wear, my opinion, based on experience, is that the rail whose hardness lies within the limits I am about to point out, will resist wear more effectually than the softer one. The maximum of rigidity, combining resistance to bending with great power of resisting shocks, has been reached in rails of the following composition:

Carbon.....	0.50 to 0.45	per cent.
Manganese.....	0.90 " 1.10	" "
Phosphorus.....	0.08 " 0.10	" "
Sulphur.....	0.05	" "
Silicon.....	0.02	" "

These rails made from perfectly sound ingots, and laid on one of the busiest portions of a great network of French railways, after *three years* of trial have not given occasion for a *single* rejection, and the wear observed has been insignificant. Of other rails made from ingots equally sound, and differing from the preceding only in having a smaller amount—from 0.5 to 0.7 per cent.—of manganese, some indeed have always been rejected after the regular test of three years, but that which has been especially remarked is that there

* Mn = 27.5, Si = 21.

has always been a notable wear of the top of the rail.

It is also known to me that the rails which have best stood the rude tests of percussion and bending, demanded by the Russian railways, contain about 0.3 per cent. of carbon, and from 1.1 to 1.2 per cent. of manganese. The manganese, without sensibly diminishing the elongation of the steel, increases its tenacity and rigidity, as well as its power of resisting shocks. It gives to the steel this grand quality of *hardness* without *brittleness*.

In the month of August, 1881, I had at my disposition quite a large number of old steel rails, made at different steel works in Germany, and taken from the railways of Alsace-Lorraine. These rails had been worn out quite rapidly; they were all in very bad condition. The oldest of them bore the date 1874, and the mark "Bochum;" the most recent came from the steel-works, "G. H. Hütte," and were dated 1879! These rails in their chemical composition corresponded for the most part with the formula of Dr. Dudley,—those, at least, which did not have any excess of phosphorus or of silicon.—but their resistance to wear has not confirmed Dr. Dudley's opinion.

I have also been able to submit to the test of a blow a rail from the Phœnix steel-works, one from the Osnabrück steel-works, and a third made by Hoesch. The Phœnix rail showed the greatest power of resistance, but the metal is soft, it changes its form considerably, and lacks in rigidity. The Hoesch rail changes in form still more, and, besides, it is brittle. It broke under the shock of 300 kilogrammes falling through $3\frac{1}{2}$ meters, the anvil weighing 12 tons (tonnes). The Phœnix rail withstood the shock of the same weight falling through $4\frac{1}{2}$ meters. The Osnabrück rail, like that of Hoesch, is brittle, but it changes its form less easily.

In conclusion, like Dr. Dudley, I am of the opinion that elements like phosphorus, silicon, and sulphur, must be reduced to an absolute minimum in a good rail. I should insist especially on the phosphorus and the silicon, and less on the sulphur, but I do not put manganese in the category of ingredients that are injurious, either to the rolling or to the use of the rail. I should give the preference to a metal containing manganese to the amount of 1 per cent. as I have indicated above, a metal which is excellent for rolling and gives a rail of superior wearing qualities.

TABLE SHOWING PARTIAL COMPOSITION OF DIFFERENT RAILS.

Description of Rail.	Mn.	C.	Si.	S.	P.
Phœnix.....	0.373	0.490	0.093	0.034	0.102
Krupp.....	0.373	0.323	0.139	0.036	0.146
Bochum.....	0.240	0.200	0.116	0.026	0.067
Union Dortmund	0.240	0.284	0.046	0.039	0.239
G. H. Hütte....	0.480	0.382	0.139	0.039	0.080
Osnabrück.....	0 586	0.170	0.466	0.045	0.174
Hoesch.....	0.453	0 330	0.291	0.038	0.119

THE STRENGTH OF BOILER FLUES.

From "The Engineer."

A PAPER on the strength of boiler flues, by Mr. W. Martin, was read before the Society of Engineers in December. The paper was in some respects peculiar. It collected and put into a convenient shape much that is not as readily accessible as is perhaps desirable. The author's purpose was to call prominent attention to the circumstance that little or no knowledge, derived from actual experiment, is in existence at this moment concerning the strength of the flues of the Lancashire and Cornish boilers. In the words of his own abstract of his paper, the only systematic series of experiments which have been made at all, were made by Sir William Fairbairn twenty-four years ago, and the formula deduced by him from these experiments is still the one in use by engineers. The trustworthiness of this formula is not, however, beyond question; and its application to different cases can be made to produce somewhat anomalous results. It cannot be denied that the results obtained by Sir William Fairbairn, consid-

ered with regard to the ignorance upon the subject prevailing at the time, were of the highest importance; and they immediately suggested simple methods of effe(tively strenghtening flues, both old and new. Nevertheless, the experimental tubes used were very unlike actual flues in construction; they were very small, and were submitted to pressure in a totally different way from that in which flues undergo strains; and since, moreover, almost all the comparable experiments were with tubes of the same thickness, it is evident that the result deduced from them cannot be regarded as final. It is much to be desired that this subject could receive a new experimental investigation, and in such a case the experiments should be with objects corresponding in shape and conditions of strain, though not necessarily in size, to actual flues. Now we have no intention to dispute the soundness of Mr. Martin's views as thus expressed, or the value of his suggestions. It is, of course, desirable that something more should be known than is known on the subject; but we may point out that any experiments made with cold water pressure would be entirely misleading, and that to carry them out in the only way that would be really useful would be very costly. What that way is we shall explain further on. Before doing so it is necessary to show why the ordinary method of testing would be useless.

It will, we think, be conceded that any experiment or series of experiments in physical science carried out under one set of conditions, can give but vague and insufficient information as to what would take place under another set of conditions. Thus, for example, to argue that because a steamship in a calm sea attained a velocity of 15 knots an hour, she could maintain a similar speed while crossing the Atlantic in the winter would be simply fatuous. The only method at present practiced for testing boiler flues consists in submitting them to cold-water pressure until they give way. To deduce from experiments such as these the behavior of similar flues when at work in a boiler is not more reasonable than would be the line of argument concerning the probable speed of a steamship which we have just considered. A powerful indirect confirmation of the truth of this was supplied on Monday night in the course of the discussion on Mr. Martin's paper. Professor Unwin said that he had carefully investigated Fairbairn's experiments, and had improved on his well-known formula by strict mathematical reasoning, and he had obtained particulars of one case of collapse, in which the result exactly coincided with his theory. "But," he went on to say, "there is a skeleton in every cupboard, and I am fain to admit that my skeleton is especially large and grissly. I have obtained particulars from Mr. Lavington Fletcher of about two dozen collapses, and not one of them agreed with my formula, the flues giving way with pressures from 50 to 80 per cent. less than I calculated." In one word, the strength of a boiler flue is in all cases less when in work than that of a similar flue tested cold with a force pump. The reason is that the flue is, when steam is up, exposed to various strains which have no existence when the force pump is used. One of the most important of these is that due to expansion caused by heat. Let us take, for example, the case of a flue 30ft. long, ½in. thick, and 3ft. in diameter. If we suppose the pressure of the steam to be 70lbs. then its temperature will be 324 deg., but the metal of the flue will be hotter than this. At the inside next the fire its temperature cannot be much less than 500 deg, while next the water it will probably be 350 deg. Here, then, we have one set of circumferential strains set up. But at a ring seam the whole of the inside lap will probably attain a temperature of 500 deg., while the whole of the outside lap will be 350 deg. The inner plates must stretch the outer, and when the boiler cools again, unless the elasticity of the outer plate is sufficient to bring it back to its original diameter, there will be a leak at the ring seam. This is no fanciful idea. It is well known that the ring seams of thick flues are a constant source of trouble unless the iron and workmanship are alike admirable. But the flue is exposed to much worse strains than these. The top is, while steam is being got up at all events, at least 200 deg. hotter than the bottom. In a length of 30ft. the top of the flue will expand until the top is nearly half an inch longer than the bottom. If it is free to do this the middle of the

length of the tube will rise, and the tube will be cambered upwards; but if the ends of the boiler are firmly tied in with gusset stays, it will not be free to rise, and a very heavy longitudinal strain will be brought to bear on the plates and seams. Furthermore, we know that no boilers are at work the surfaces of the flues and furnaces of which are quite clean; on the contrary, they always have a coating of scale on them of greater or less thickness. The result of this is that the flues must be of necessity very much hotter than they would be otherwise; and, in point of fact, no one knows how hot a furnace crown is when a boiler is at work. But it is very well known that as the temperature of iron is augmented its stiffness greatly diminishes, and inasmuch as flues are never circular, they depend for their strength very much on their stiffness, and if this be impaired the flue is weakened. This point is not so fully understood as it ought to be. We may regard any section of the circumference of a flue as an arch, built up of practically incompressible voussoirs. If these voussoirs were united in such a manner that they could not move on each other, then we should have a rigid arch, which could only fail by the compression of the material of which it was made; but if the arch was not so tied together, then it would remain free from deformation only so long as the line of pressures fell between the extrados and the intrados of the arch—that is to say, within the thickness of the plate. Now the comparatively thin plate of a flue can be bent, and if any of our readers will take the trouble to get out on his drawing board a section of the circumference of a flue, he will see that the bending in or out of that section is precisely analogous to causing the rotation of voussoirs on their inner or outward edges. In other words, the fact that the iron can be bent is analogous to the use of a weak or flexible cement to unite voussoirs. This being the case, it becomes evident that the more readily the iron can be bent the weaker is the arch. This would not be true if the flue were a perfect cylinder, for then the line of pressures must fall in the thickness of the plate. But it is easy to see that in the case of, say, a 3ft. flue, $\frac{3}{8}$in. thick, a deformation to the extent of but $\frac{3}{8}$in. would throw the line of pressures within or without the plate. All flues with lap joints are out of truth, and were it not for the stiffness which the lap imparts, and the fact that the thickness being doubled at the lap the line of strain still falls within the metal, such flues would be much weaker than they are. If our reader has followed us thus far, they will see that the heating of a flue by depriving it of its stiffness has just the same effect as the weakening, let us say, of the mortar in a brick arch; and it must be clearly understood that this has nothing at all to do with the ultimate strength of the iron. That may or may not be greater at the temperature really attained by the flue than it is at any lower heat. If, then, it can be shown that iron is less able to resist bending strains when hot than when cold, it follows that comparatively moderate increases in temperature may much weaken a flue. It is probable that at a temperature of about 800 deg. the stiffnes of plate iron, in other words the resistance which it can offer to bending strains, is reduced by one-half; and if this be so we have at once an hypothesis which will explain many cases of collapse now regarded as mysterious. A moderate thickness of scale will permit the temperature of a furnace crown to rise much above that of the water. The overheating may not be sufficient to leave any mark on the iron, nor may it suffice to reduce the tensile strength of the metal to any appreciable extent, but it will take the stiffness out of it, and then a collapse may ensue. It is only necessary to establish the soundness of these propositions—(1) that the flue should not be quite cylindrical, and (2) that it is easier to bend a bar of iron when it is hot than when it is cold, and we do not think this can be disputed.

If, then, we desire to obtain any definite information concerning the strength of boiler flues, they must be tested under conditions similiar, as far as possible, to those which obtain in practice. In the United States and on the Continent, in a few cases tests have been made by filling up the boiler as full as it will hold with cold water, and then lighting a small fire in the furnace; the water will then rapidly expand, and while still nearly cold will put on the requisite pressure. Such testing requires to be very carefully done or the boiler may be irreparably injured,

and the system has the disadvantage that the test is still a cold water test. The method of experimenting on boiler flues which we advocate is, so far as we know, new. The flue must be fitted in a boiler shell having a very much higher bursting pressure than the collapsing pressure of the flue. Let this shell be filled up with cold water within a few inches of the top, and let the fires be lighted and hard pushed, just as they would be in practice in getting up steam, and as soon as the water has reached 212 deg. let the force pump be put on, and the flue tested to destruction, the fires being left burning all the time. The boiler being in a suitable building, with no one in it, no accident would occur when the boiling water began to run out. A large force pump should be used, competent to run up the pressure to the required point in a couple of minutes at most, and to crush the flue, even after ring seams had been started. It would occupy more space than we can spare to describe minutely all the precautions which would be necessary to secure accurate results. They will suggest themselves, we venture to think, to any one who has had fair experience in testing boilers. In this way, and in this way only, can flues be tested under nearly the same conditions in which they are strained im boilers; and if the experiment is ever carried out, we can promise that the result will be found interesting, if not startling.

REPORTS OF ENGINEERING SOCIETIES.

AMERICAN SOCIETY OF CIVIL ENGINEERS. —Meeting of December 6, 1882. Vice-President William H. Paine in the chair. John Bogart, Secretary.

The supply of water for cities and towns from subterranean sources, or ground water, as developed in the United States since 1870, was described by Mr. J. J. R. Croes, M. Am. Soc. C. E. It was at first supposed that such supply could be obtained by filtration of river or lake water through the gravel of its banks. It was discovered, however, that in fact, much more water came from the land side than from the river, and that wherever such a source of supply is successful, the water really comes from the underground reservoirs or streams which are found generally in all valleys containing much gravel. The wells, galleries and basins constructed in various places, were described, and their success or failure indicated. It was stated that experience was generally against the construction of open galleries or canals on account of the vegetable growth which always occurred in such cases. The paper was discussed by members present, and will be published in the transactions.

December 20, 1882.—Mr. William P. Shinn, M. Am. Soc. C. E., read a paper on the "Increased Efficiency of Railways for the Transportation of Freight."

The first portion of this paper gave, from carefully gathered statistics, a valuable amount of information in regard to the actual increase of traffic upon American Railways. In 1860 the tonnage mileage of the New York Central and Hudson River Railroad, the Erie Railway, and the Pennsylvania Railroad was about equal, and amounted in the aggregate to a little over three-fourths of that of the New York State Canals, and in 1870 each of these railroads averaged about the tonnage of the canals, and in 1880 they averaged each nearly double that of the canals.

The aggregate tonnage mileage of the other railroads was, in 1881, 1,217 per cent. more than 1860. Statistics were also given showing the increase of population of railroad mileage, of the production and export of grain and other leading exports. The means by which this rapid increase of freight transportation had been developed was considered under two general heads, namely, improvements in the physical conditions of the railroads, and improvements in the administration. The improvements in the physical condition were treated on under these heads:

1. Improved track or "permanent way," including bridge structure.
2. Additional sidings, and second, third and fourth tracks.
3. Increased capacity and strict classification of locomotives.
4. Increased capacity of freight cars.
5. Additions to terminal facilities.

The improvements in the administration were referred to under the following heads:

6. Improved methods of signaling.
7. Running locomotives "first in first out," and running freight trains at higher rates of speed.
8. Consolidation of connecting lines under one management by purchase, lease, amalgamation or otherwise.
9. Running freight cars through from point of production to tide-water without trans-shipment.
10. Issuing through bills of lading (or freight contracts) from western points of shipment to Atlantic and European ports.

The general introduction of steel rails was stated to be the very corner stone of increased efficiency. The improvements in all the directions referred to were treated of and described at considerable length.

The second portion of the paper presented the views of the writer as to the means whereby still greater efficiency could be most economically obtained. The constant demand is for more transportation facilities, for more cars. In the opinion of the writer, what is needed is not so much more cars, as more movement of cars. Freight blockades will be prevented, not by having more tracks to stand cars upon,

but by having fewer standing cars. It was shown that upon one railway there had been a decrease in the miles run by the cars of 21 per cent. between 1868 and 1881—and that the Union Line cars between 1879 and 1882 were increased 49 per cent. in number, while the mileage run by them decreased 16 per cent. in the same period. The remedies suggested by Mr. Shinn were, more main tracks, more locomotives, more trains, the improvement of the making up of trains at the points where cars are loaded. The detention of cars at stations and private sidings, and the absence of cars on foreign railroads were considered as among the greatest causes of loss, and the writer suggests that the remedy will be, to charge a per diam charge for cars when on foreign roads, and that this charge should be based upon the average economic value of its cars in use to their owners.

The paper was discussed by Messrs T. C. Clarke, G. S. Greene, Jr., W. C. Andrews, C. Macdonald, C. E. Emery, and by the author.

By a vote of the Society it was directed that this paper should be discussed at the annual meeting. Members of the Society and others interested in this subject are requested to contribute to this discussion. The annual meeting of the Society will occur January 17 and 18, at the Society house in New York. The first session of the meeting will be at 10 A. M., January 17, 1883.

ENGINEERS' CLUB OF PHILADELPHIA, December 16th, 1882. Mr. Henry G. Morris in the chair.

Mr. E. F. Loiseau, introduced by Mr. Washington Jones, read a paper on the subject of his Artificial Fuel, which he exhibited, in process of consumption, in the Club room grate. After giving a short historical sketch of the manufacture of artificial fuel in Europe, where "briquettes" have been made for years past, Mr. Loiseau said that the aim of inventors has been to manufacture small lumps in paying quantities and, so far, the attempts have been failures. Sixty-eight fuel factories are successfully operated in Europe, but they all make brick-shaped lumps, too large for family use. Mr. Loiseau claims to have solved the problem, and states that the Loiseau Fuel Company, at Port Richmond, cannot supply the rapidly increasing demand for the small egg-shaped lumps (weighing about 2 oz.), which they manufacture at present.

Mr. Ashburner presented the following. I desire to call the attention of the Engineers' Club to the first finished maps and sections of the Anthracite Survey (2d Geol. Surv. of Pa.), and to state briefly the general plan of publication which has been adopted. Thirteen sheets have just been engraved and printed, to illustrate the mining and structural geology of the Panther Creek basin, which lies at the extreme eastern end of the Southern Coal Field between the Little Schuykill River at Tamaqua and the Lehigh River at Mauch Chunk. These illustrate the character of the charts which will be constructed in the other parts of the region.

The basis for the work is the surveys of the corporate and individual coal operators in the region, which have been made with so much care and precision. These are afterwards connected and extended by the field work of the Geological Survey corps. The charts are to be published of uniform size (26 by 32 inches,) and on the following scales:

1. Mine maps, showing the mine workings and the structure of the coal-beds, by underground contour curves 50 feet vertically apart. Scale 800 feet=1 inch.

2. Topographical maps of the surface of the coal basins, in contour curves 10 and 20 feet vertically apart Scale 1,600 feet=1 inch.

3. Vertical cross sections of the coal basins. Scale 400 feet=1 inch.

4. Columnar sections of the coal measures, showing the relation of the coal-beds and the character of the rock intervals. Scale 40 feet =1 inch.

5. Columnar sections of the individual coal-beds. Scale 10 feet=1 inch.

These sheets will be supplemented by others of a miscellaneous character.

Prof. L. M. Haupt presented a short description from Mr. John C. Trautwine, Honorary Member of the Club, and Mr. W. E. Babbit, C.V.R.R., of the floating drawbridge at Rouses Point, and also a drawing, with notes, of a wooden arch bridge across the Genesee River below Rochester, of 352 feet span, 54 feet rise, which failed by descending at the haunches and rising at the crown.

ENGINEERING NOTES.

THE GREAT WALL OF CHINA.—An American engineer, who, being engaged in the construction of a railway in China, has had unusually favorable opportunities of examining the famous Great Wall, built to obstruct the incursions of the Tartars, gives the following account of this wonderful work: The wall is 1728 miles long, 18 feet wide, and 15 feet thick at the top. The foundation, throughout, is of solid granite, the remainder of compact masonry. At intervals of between 200 and 300 yards towers rise up, 25 to 40 feet high, and 24 feet in diameter. On the top of the wall, and on both sides of it, are masonry parapets, to enable the defenders to pass unseen from one tower to another. The wall itself is carried from point to point in a perfectly straight line, across valleys and plains and over hills, without the slightest regard to the configuration of the ground; sometimes plunging down into abysses a thousand feet deep. Brooks and rivers are bridged over by the wall, while on both banks of larger streams strong flanking towers are placed.

THE canal of the Isthmus of Corinth, which was commenced on the 2nd May, 1882, with 800 men, mostly Italians, and which is to be finished in five years, will be like the Suez Canal, about 73ft. broad and 27ft. deep. For ships from the Adriatic 1fr. per ton will be charged; for ships from the Eastern Mediterranean 50c. per ton; for passengers, 1fr. each.

THE projected tunnel under the St. Lawrence, at Montreal, is to be 16,000ft. long, or almost exactly three times the length of the Hudson River tunnel, and thirteen and one-third times that of the Thames tunnel. The greatest depth will be 166ft. below the entrances. The grades are not as steep as those of the Hudson River tunnel. The contract requires the completion of the work (by Mr. Romillard) in three years.

RAILWAY NOTES.

RAILWAY SPEEDS.—A German journal, *Die Verkehrszeitung*, has recently published a tabular statement of the maximum speeds of trains in Great Britain, Europe, and the United States. The table is inaccurate and incomplete to a very considerable degree, but it contains nevertheless some suggestive information. The American *Railroad Gazette* has supplemented it by a statement of the speeds of some of the fastest trains in the United States, and this statement has been in turn supplemented and corrected by two correspondents. The prominent fact which comes out is that the time taken to traverse any given distance between two places connected by rails shows that speeds are much slower than most people suppose. If we take, for instance, the run from London to Edinburgh, a distance of 397 miles *via* York, this is made in nine hours by Great Northern trains, the average speed being thus 44.1 miles per hour. From Euston the distance is 401 miles, and London and North-Western trains make the run in 10 hours, or 40.1 miles an hour. By the Midland Railway the distance is 404 miles and the time 10 hours 5 minutes, or very nearly the same speed. The German journal to which we have just alluded makes the distance 116 miles and the speed 41 miles an hour, which is an error. Some of the fastest trains in the world are those run between Leeds and London. From King's Cross the distance by the Great Northern is 186¼ miles. From St. Pancras by the Midland it is 196 miles. The fastest train on the Great Northern makes the run in 4 hours 30 minutes, giving an average velocity of 43.5 miles an hour. The fastest train in the world is the Flying Dutchman, broad gauge, which makes the run to Swindon at 53¼ miles an hour. The Great Northern trains run from London to York, 188 miles, at 48 miles an hour, and at least one train runs to Peterborough at 51 miles an hour. The run from London to Grantham has been made repeatedly al 51 miles an hour. On the United States railways the quickest time appears to be that made between Jersey City and Philadelphia, 89 miles, made at the rate of 47 2-3 miles an hour. There is not in the world a train timed to run at 60 miles an hour, although it is, of course, certain that that velocity is often exceeded. If a speed of 60 miles an hour could be maintained continually between London and Edinburgh, the journey would occupy only 6 hours 36 minutes; and allowing for three stops of ten minutes each on the route, the time would be under 7¼ hours, instead of ten hours. So far as the machinery of a railway is concerned—by which we mean the road, the rolling stock, and the signals—there is nothing to prevent an average speed of 60 miles an hour being maintained. That it is not attained is certain. It is worth while to inquire why.

The first essential to great average speed is that the runs shall be long; that is to say, there must be long intervals between stopping places; and this is necessary not so much because of the time lost at a station, as that spent in getting into and out of it. The station must be approached and left at a comparatively slow speed if for no other reason, than because it would be dangerous to do otherwise. The next essential is that the train shall be light, because the engine ought to be able to maintain a high speed when running up hill as well as when on a level. High speed trains cannot fully avail themselves of the compensating effects of inclines. Let us suppose that the maximum speed attained must not exceed 70 miles an hour. Then if the up and down inclines balance each other, the speed of the train must never fall below 50 miles an hour or else the average velocity cannot be 60 miles an hour. If the maximum speed permitted was 60 miles an hour, and the average speed 40 miles an hour, then trains might ascend the banks at 20 miles an hour, and still keep time. It would not be advisable to run at a higher speed under any circumstances with ordinary rolling stock than 70 miles an hour, and there are no locomotives built which would never run at less speed than 50 miles an hour between London and Edinburgh with a greater load than about 55 tons; that is to say, 75 tons for engine and tender, and 45 tons for four passenger coaches and a brake van, or in all 130 tons. The maximum number of passengers carried would be about 200, so that the dead weight moved would be 13 cwt. per passenger. The first class fare now is 57s 6d. If we call it £3 we should have £600 as the total returns, or, say, 30s. per mile run. It is not at all impossible that such a train might be made to pay. No doubt there would be now and then a strong temptation to add another carriage, but to give way would be to ruin the whole scheme. Its success would depend entirely on keeping the load to be hauled so small that engines might be worked at a speed never under 50 miles an hour. Three engines would be employed on the run, each making a distance of about 135 miles; and we cannot see that any great difficulty would be encountered in doing this. It must not be forgotten, however, that although the load would be small, the excessive speed would make large demands on fuel and water. It would not be safe to reckon on a less consumption of coal than 40 lbs. a mile, so that the tender would have to carry about 3 tons; and allowing that each pound of coal evaporated 10lbs. of water, including waste, the tank must hold about 5,400 gallons, or 24 tons of water. It would, however, be highly objectionable to attempt to carry such a load as this, and Ramsbottom's water troughs supply a way out of the difficulty, and a tender capacity of 2,000 gallons would be ample. On a grate of 20 square feet the consumption would be at the rate of 120 lbs. an hour, which is much too fast for comfortable or regula-

working, and a special design of engine would be absolutely indispensable, one with a very long grate being required to provide the requisite surface. It will be seen that the engine which could comply with the required conditions is yet to be designed. The fastest train in the kingdom is, as we have said, the Flying Dutchman, and this is 7ft. gauge. Many persons think that the utility of the wide gauge in this case is that it prevents risks of oversetting. This is an entire mistake. There is no speed, perhaps, at which a train can run which would entail the least risk of such an accident on a good road of 4ft. 8in. wide. The speed advantage conferred by the 7ft. gauge is that it permits an enormous boiler—and particularly an enormous grate—to be used. The Great Britain, for example, has nearly double the heating surface of powerful express engines on other lines, and there would be no difficulty in increasing its grate surface up to 35 square feet, if necessary. Engines of this class have 1,953 square feet of heating surface and 21 square feet of grate. The Flying Dutchman consists of engine and tender weighing 65 tons 18 cwt., one eight-wheeled van 18 tons, and five eight-wheeled coaches weighing 88 tons; total, 169 tons 18 cwt.

The principal reason why high speed trains are not run is that it is difficult to provide locomotives of sufficient steaming power. It is evident that if an engine could be produced which could do as much work with 20 lbs. of coal per mile as the existing engine does with 40 lbs., an enormous advantage would be gained; but it is not at all probable that any great saving can ever be effected in the consumption of coal. Mr. Webb's compound engine may do something, but the existing locomotive is "bad to beat" in this respect. The designing of a locomotive to make long runs at 60 miles an hour average speed presents, however, an interesting problem, which will, perhaps, be attacked some day. We have pointed out briefly a few of the more important conditions to be kept in mind. Locomotives are so absolutely interwoven in the dimensions of parts, if we may use the words, that to augment the area of a grate by one-half will suffice to almost revolutionize the proportions of an engine. That a good engine of this kind could be made we have, however, no doubt. It remains to be seen whether England or America will have one first. Every year the number of miles travelled by each member of the population increases, and this growth of travel will no doubt rapidly encourage the running of long distance express trains, by which alone can high mean speeds be attained or made to pay.—*The Engineer.*

THE Prussian Minister for Public Works has given orders that cast steel rails should be laid in the various curves which occur on the newly-opened Stadt-Bahn, or local railway of Berlin. The *National Zeitung* states that on account of the heavy traffic and consequent wear of the rails at such points, it has been ordered that new rails are to be laid every three months at these portions of the line. If this is correct, either the rails must be very poor or the traffic is different to that anywhere else on the Continent.

MESSRS. VAN DER ZYPEN BROS., of Deutz, near Cologne, who employ 120 men in the smithy, are turning out railway wheels by the hydraulic press at the rate of 15,000 a year, finished and mounted, for Germany, Russia, Italy, Turkey and Australia. The spring ring is generally used for fixing the tire, and it is exclusively adopted by the German Government. The firm is now giving special attention to the manufacture of a carriage wheel like the Mansell, but of compressed paper, which is found to give good results.

ORDNANCE AND NAVAL.

DISCOID PROJECTILES FOR CANNON.—Before the adoption of rifled cannon with oblong projectiles, several eminent artillerists conceived the idea that good effects might be obtained with a disc-shaped projectile, having a rotation about its axis of figure, and fired so as to present its equatorial surface to the air. Experiments were indeed made in France, not very long ago, with a portable arm and discord projectiles. The chief advantages hoped for were diminished air resistance, great stability of the axis of rotation, and consequent accuracy, penetrating power, etc. As these advantages were largely had with oblong projectiles, the matter dropped. In the *Revue d'Artillerie*, however, Captain Chapel has lately called attention to a new property of these discoid projectiles, and thinks it might, in some cases, prove valuable. This a tendency in the descending path, *to return*, and to strike the ground at an angle above 90 degrees. In this way, artillery placed opposite a line of defence would be able to strike the defenders from behind, and the ordinary methods of defence would require modification.

ARMOR-PLATE TESTS.—We learn from the *Times* that Messrs. John Brown and Co. (Limited), Atlas Steel and Iron Works, Sheffield, have received orders for the entire armor for a new war-ship, to be called *Dmitri Domskoy*, to form one of a series of vessels to be constructed next year. The *Dmitri Domskoy* will not be an exceptionally powerful man-of-war, but the Russian Government are now making inquiries with regard to several very large ships which are to be commenced in 1883. The Sheffield manufacturers of compound armor expect that the results of the trial at Spezzia will be in their favor, and the order for the *Lepanto*—the sister ship to the *Italia*—is looked for very soon. One result of the trials will be the use of a larger number of bolts in the plates than have hitherto been used. The French all-steel plate, after it was smashed, kept its pieces together by means of its bolts, while the Sheffield plates fell away. It is obvious that the plates, if kept together on the ship's side, would still afford some protection against further shots, though there is no doubt that the additional bolts will weaken the armor. Austria is also making inquiries for armor on the compound

system. The plates for Russia will be of the "Ellis" type. The other manufacturers, Messrs. Charles Cammel and Co. (Limited) completed some time ago the plates ("Wilson" patent) for the Russian vessel *Vladimir Monomach*, of which the *Dmitri Donskoy* is the sister ship. Messrs. Cammel and Co's plates came very satisfactorily out of the tests at St. Petersburg on the 24th ult. Two armor plates, 12 inches thick, each weighing about 12½ tons, were tested. One of the plates was made of steel by Messrs. Schneider, Creusot Co., France, and the other was on Wilson's compound system, steel-faced, made by Cammel and Co. The gun used was an 11-inch Aboukof breechloader, range 350 feet, chilled cast-iron projectiles, each weighing 615 lb. The first snot, which had a striking velocity of 156 feet, broke the creusot plate into five pieces, the penetration being 13 inches. The second shot penetrated 16 inches, and broke the plate into nine separate pieces. The third shot still further broke up the plate, and carried away fully one-third of it. the projectile penetrating the whole tar get, and was found 740 yards to the rear, being apparently uninjured. The Wilson (Cammel's) plate was then tested under similar conditions. The first shot produced only a few slight cracks on the steel face of the plate, while the projectile was completely broken up, a small portion of the heading remaining in the plate. The penetration could not be obtained, but was believed to be 4 to 5 inches. The second projectile produced only a slight penetration with no preceptible damage to the plate itself. The third shot was not fired, four of the bolts which held the plate being broken, and it is necessary to replace them before the experiment can be completed. It is clear from these trials, as well as those at Spezzia, that the Creusot Company are wise in using a large number of bolts to keep the plate together after it is partially destroyed. The result, so far as the test has gone, is very favorable for the Sheffield plate.

ARTILLERY TRIALS AT SPEZZIA.—Some experiments are being carried out at Spezzia with the 100 ton muzzle-loading gun which was fired last week at a compound plate made by Cammel, a compound plate by Brown, and a steel plate by Schneider, of Creusot. All three plates were of the same thickness, viz., 19 inches, with 4 feet of wood backing. The velocity of the shot was calculated to perforate a wrought-iron plate of the same thickness. According to the *Army and Navy Gazette*, the Cammel plate was not perforated, but the corner of it was separated by cracks through the whole thickness. The Schneider plate was less penetrated than the Cammel, and no cracks were apparent. The Brown plate was less penetrated than the Schneider, and only cracked in the steel facing. The wrought iron acted as a cushion; it bent a little, and so absorbed the force of the blow. On the experiments being continued, the first round was fired at Brown's plate, which broke into four large pieces and many small pieces. About three-quarters of the plate fell from the target, and the backing was much damaged. The second round was at Cammel's plate, which broke into five large and many small pieces, and fell completely from the target. The energy of the projectile in both rounds was about 33,900 tons. The plates were not penetrated in either case, but dashed to pieces. A ship would leak through the injury caused to the backing. The Whitworth shot fired at the Schneider plate failed to penetrate more than 8 inches, but completely broke up the plate, and drove in the backing. The Gregorini steel shot, fired afterwards, left the Schneider target a mass of ruin The results of these experiments are considered to reopen the whole question of naval gunnery and armor. The gun has not yet been fired with its full battering charge.

MR. JOHN HAUG, in presenting the Engineers' Club of Philadelphia with a copy of Lloyd's Rules for Iron Ships, stated the number and tonnage of ships built in Great Britain in 1881 as follows:

	Built in 1881.			Lost in 1881.	
	Number.	Tonnage.	Material used.	Number.	Tonnage.
Steel steamers.....	34	68,366	32,000 tons.	1	1,536
Steel sailing vessels....	3	3,167	1,500 tons.		
Iron steamers.....	411	590,503	300,000 tons.	139	138,370
Iron sailing vessels....	50	68,650	34,000 tons.	52	43,936
Wooden steamers ..	30	1,659		18	1,704
Wooden sailing vessels.	259	16,448		821	168,579
Total.....	787	748,793		1031	354,125

The principal changes in the rules for 1882 have been in water-tight bulkheads, of which more are now required in longer and larger vessels, and they are to be extended to the principal upper deck.

Vessels of extreme proportions (over 11 depths in length) have to be better strengthened in their top and bottom members, by doubling strokes, &c.

Treble rivetted buttstraps are required to a greater extent, as forming stronger joints.

The rules for boilers, machinery, pumping arrangements, spare parts for machinery, &c., have been extended and improved, with a view to greater safety at sea.

As ship-building of steel is increasing, a reduction of 20 per cent. from the scantlings required for iron is permitted, giving ships so much more carrying capacity. A complete set of rules for testing all materials, insures uniform quality in steel used. Steel castings, by the Siemens-Martin or Bessemer process are also now used, in place of large and expensive forgings, for stemports, rudders, stems, &c., and they have been found strong and tough; they are less expensive than scrap-iron forgings, and the risk of bad welds and inconvenience of rough and uneven shape is avoided.

The latest circular issued by Lloyd's Register offers to fix a proper load line for each vessel, according to its style, form, &c.; thus the rules not only provide for its proper strength, but

also its sea-going qualities, &c. This is of the greatest importance, in view of the many disasters that have occurred from the want of those qualities.

Mr. Haug also exhibited and described drawings of his own and other valve gears.

The one patented by Mr. Haug belongs to that class in which the longitudinal and lateral motion of an eccentric, or other rod, are both utilized for obtaining a variable cut off and reversing motion. In comparison with F. C. Marshall's and Joy's valve motions, which have both been extensively used, this valve motion consists of fewer parts and joints, can be more compactly arranged, and has the least amount of motion to all its parts, being thus more accessible for oiling, &c. Compared with the link motion, it is considerably simpler, has less friction (all sliding friction being replaced by bolt friction), requires less force to reverse, and will consequently throw less strain on all its members.

IRON AND STEEL NOTES.

STEEL IRON.—The question of producing a metal possessing the physical properties of both iron and steel has for some time past received attention at the hands of practical metallurgists and others. One of the latests workers in this direction is Professor M. Keil, who has succeeded in producing a compound metal which is stated to possess the characteristics of both metals. The professor, in giving his experiences on the subject states that the difficulties can be obviated only if the two materials are intimately united into a whole. After many experiments, success has, it is claimed, at last attended them, and a material has been produced answering every requirement, and to which the name of "steel-iron" has been given. The following five descriptions have been made :—(1) steel by the side of iron ; (2) steel between two layers of iron; (3) iron between two layers of steel; (4) the core of steel, the surrounding shell of iron; (5) the core of iron, the surrounding shell of steel. This steel-iron is manufactured in the following manner. A cast-iron mould is divided into two parts by a thin sheet of iron securely fixed in it. The fluid steel, as well as the fluid wrought iron, which have been freed before smelting from substances preventing welding, are poured at the same time, and in the same quantity, into this double mould ; the separating plate serving as the medium welding both parts, steel and iron, completely together, so that they form an inseparable whole. The plate serves as a separator and a welding agent at the same time. The success of the operation depends upon the quality and the thickness of the plate. The latter must be of a certain thickness, to prevent the two glowing and liquid masses burning through it ; and it must not be too thick, so that they are able to bring it up to welding-point while rising in the mould. The dimensions of the plates depend upon experience, and, naturally, are regulated by the dimensions of the castings. The manufacture of the above-mentioned five kinds are the same in principle. In numbers 2 and 3, however, the mould is divided into three equal parts, by two strips of plate ; in numbers 4 and 5 the core is formed by a sheet-iron pipe standing in the middle of the mould. It is stated by Professor Keil that the product thus obtained may be used for a great many purposes. Steel upon iron will be useful for rails, armor plates, and anvils, the hard steel face reducing wear and tear, and also, as in the case of thief-proof safes and armor-plates, withstanding the attacks of even the hardest drill, while the iron prevents cracking consequent upon heavy blows. Parts of machinery and tools which are subject to powerful pressure, and are exposed at the same time to great vibration, are best made of the material with tough core and hard surface. The wear and tear would be slight, while the soft core imparts considerable strength and prevents fractures. From what has been said respecting the quality of this description of steel-iron, it will be seen that the extent of its application promises to be a wide one, partly on account of its undoubted excellence, partly also, from its many qualities, because it may be used for a great variety of manufactures.

THE DEPHOSPHORIZATION PROCESS.—It may be said that the problem of manufacturing steel from phosphoric pig has been solved, both in the converter and reverberatory furnace, by the employment of a magnesian lime lining. The elimination of phosphorus is by this means as satisfactory as possible, that of silicon is nearly complete, and even sulphur is removed to a large extent. From comparative analyses made in August and September, 1881, it is shown that basic steel is purer than acid steel, and has a more uniform composition. The tensile mechanical tests show that the results furnished by the basic Bessemer steel are sensibly more regular than those given by acid steel. Rails manufactured from these two varieties of steels give similar results under the static and dynamite tests. The inconvenience caused by the presence of blisters on the ingots, met with at starting the basic process, has been overcome by raising the temperature of the metal at the moment of pouring. The state has therefore been led to accept indifferently rails, made from either variety of steel. In the reverberatory furnace, the manufacture of basic steel is more easily carried out than in the conventer, and the dephosphorization is more complete. Metallurgists, therefore, are now in possession of two different methods of manufacturing steel, both in the converter and in the reverberatory furnace. The one consisting in treating pure products in an apparatus furnished with a siliceous lining ; the other in treating impure products in presence of a basic lining. The following question, therefore, naturally seems to present itself. Since, other things being equal, linings of magnesium lime offer a more complete purification than siliceous ones, why not treat pig metal, even that resulting from pure ores, in a basic apparatus ? By this means these latter should give steel of very great purity. So far as concerns the reverberatory furnace, it appears probable that

in the majority of cases the siliceous hearth will be replaced by one of magnesian lime; the carrying out of a basic operation presenting, as we have explained above, no difficulties. But with the converter the case is different; it would not be found advantageous to treat in a basic-lined converter metal such as that used in the acid process, since this metal is high in silicon; and, as we have shown above, this circumstance presents a grave obstacle. On the other hand, if the blast-furnace charges were so regulating that the resulting pig was but slightly siliceous, there would probably not be sufficient heat generated by the molecular combustion to assure the liquidity of the bath and slag. The treatment of pure metal, therefore, in the basic converter, is not without drawbacks. It might, it is true, be arrived at by the employment of the transfer method; that is, by removing silicon in an acid vessel, then running the metal into a basic lined vessel, and there finishing the blow. But this process offers the great drawbacks of being expensive and complicated. We will not, therefore, enlarge upon this question, the solution of which belongs to the future.

THOMAS-GILCHRIST PROCESS.—There has been much discussion this week upon the iron exchanges in the Midlands, touching the relative merits of steel and iron and the progress of the Bessemer basic method. The twenty-five converters now being erected upon the Continent and the nine in England will, when at work and added to the converters now in operation, bring up the production to 1,196,600 tons per annum. Such an augmentation upon the current yearly continental and home make, which reaches 572,604 tons, should be encouraging to the patentees, while it certainly gives much additional value to the poorer iron-yielding ores wherever they are found. Already the tap-cinder of the ironworks is worth more money; and in the United States accumulations are being stored at a figure which it would never before have been thought such bye-products would realize. On 'Change in Wolverhampton on Wednesday, Mr. Alfred Hickman, blast furnace proprietor, who has taken up the system there with spirit, stated that he was about to erect two more furnaces at Spring Vale, close to the land selected for the works at which the Staffordshire Company will make basic-Bessemer steel. Mr. Joseph G. Wright, the chief proprietor of Monmoor Ironworks, in the same town, who had taken part in the discussion last week upon the paper which Mr. Gilchrist read at Dudley, confirmed on 'Change his statement at the formal discussion, that the cost of puddled bar in Staffordshire made from pig at 42s. per ton was 80s., and held that whatever might be the cost of the raw ingot, it would require 5s. 6d. a ton to be spent upon it before it was brought into an equally forward state with the puddled bar for subsequent manufacture. This computation leaves the difference in favor of crude steel from the same class of pig at 4s. 8d per ton, assuming Mr. Gilchrist to be correct in his estimate of 69s. 8d. as the cost of the ingot; and reduces Mr. Gilchrist's estimate in favor of steel by only 7d. per ton. But Mr. Wright is not, at the present stage of his knowledge of the capability of basic ingot iron, prepared to admit that that material would be equal to the puddled bar in in all-around value. The operation of the Staffordshire Steel and Ingot Iron Company will supply the ironmasters of the country with the precise statistical facts which they seek before they can make up their minds upon the merits of ingot iron is a raw material not for rails and ship-plates and sheets alone, but also for the general run of sorts demanded by the merchant. Meanwhile it is interesting to note that by the formation of "The Dephosphorizing and Basic Patents Company, Limited," with a capital of £30,000, for the purpose of purchasing "letters patent granted respectively to Messrs. Sidney Gilchrist-Thomas, G. J. Snelus, E. Riley, P. C. Gilchrist, and H. C. G. Harmet, for inventions relating to the manufacture, purification, and dephosphorization of iron and steel, and the manufacture of furnace lining and lining material," the several patentees of the Thomas-Gilchrist process would seem to be adjusting in a business-like fashion the ratio of their respective claims to the emoluments of the combined invention.

BOOK NOTICES

PUBLICATIONS RECEIVED.

MONOGRAPH OF THE CENTRAL PARTS OF NEBULA OF ORION. By Edward S. Holden. Washington: Government Printing Office.

TRANSACTIONS OF THE AMERICAN INSTITUTE OF MINING ENGINEERS. Report of the Colorado Meeting.

THE NAVAL USE OF THE DYNAMO MACHINE AND ELECTRIC LIGHT. By Lieut. J. B. Murdock, U. S. N.

REPORT OF THE CHIEF SIGNAL OFFICER FOR 1880. Washington: Government Printing Office.

A SCHOOL COURSE ON HEAT. By W. Larden, M. A. London: Sampson, Low & Rivington. Price, $2 00.

The scope of this work is so fairly stated in the preface, that an abstract from the latter will suffice to explain the character of the treatise:

"It is assumed that the reader has no previous knowledge of physical science at all, but is being introduced to it for the first time. Hence the reasoning and explanations are at first very elementary in character, greater brevity being only gradually attained.

"For the same reason the writer has thought it well to introduce matter not actually belonging to the subject of heat, wherever such introduction seemed necessary to the clear understanding of any matter that does belong to this subject.

"The mathematical parts are treated in a very careful manner. At the end of each chapter are given questions on the subject of that chapter."

To which we may add that so far as we know it is the book best calculated to satisfy the young artisans and engineers who wish a reliable aid to their efforts to supplement their school course of study.

THE TRANSIT INSTRUMENT. By Latimer Clark, M. I. C. E. London: Published by the author. Price, $2.00.

This is designed for amateurs who are inclined to practice with the astronomical transit in the determination of time.

The theory of the problem is clearly stated, and all necessary instructions are given for the practical solution. Diagrams descriptive of the construction and adjustments of the instrument are given.

Tables giving the Greenwich time of transits of the sun and of certain fixed stars are given at the end of the volume.

The book will prove valuable to gentlemen of leisure and of scientific inclinations.

EXAMPLES OF IRON ROOFS. By Thomas Timmins. Vol. I. London: Thomas Timmins. Price, $5.00.

A thin quarto volume of folding plates containing designs of iron roofs drawn to scale. The details of intersections are also given to an enlarged scale.

A graphic solution of each form is also given on a single plate.

Tables of strengths of columns and struts are also added.

The whole constitutes a convenient guide to the builder who desires to construct either of the given forms.

LIFE OF JAMES CLERK MAXWELL. By Lewis Campbell, M. A., LL. D., and William Garnett, M. A. London: Macmillan. Price, $6.00.

This volume of 661 pages is in three parts, of quite unequal interest to the average reader. The first part presents the biography; the second, the contributions to science, and the third the original poems of Mr. Maxwell written at different times between 1844 and 1878.

The biographical portion, of course, presents outlines of some of the earlier scientific papers, partly by way of letters and partly as a narrative, of the growth and development of the hero.

There is much of the first and second parts that scientific readers will find intensely interesting. There is enough in the third portion to demonstrate that the subject of this interesting memoir was not a result of one-sided development. The book is handsomely printed and is well illustrated.

MISCELLANEOUS.

THE ORDNANCE SURVEY OF SCOTLAND.—It has just been announced that the Ordnance Survey of Scotland and the adjacent islands, after being in progress over thirty-seven years, has been entirely completed. In order that this important work might be overtaken within a reasonable period of time, Scotland was divided into three sections; and during the last few years the average number of men employed upon the work has been from eighty to ninety. The bulk of the surveyors were civil assistants, but the superintendents of the survey were officers of the Royal Engineers—the Edinburgh division having been under the command of Captain Kirkwood during the past five years. With the exception of the counties of Edinburgh, Kirkcudbright, Wigtown, Haddington and Fife, which were surveyed and published on the 6 in. scale before the larger scale was adopted, the whole of the agricultural districts have been surveyed, and the results are now published on the scale of $\frac{1}{2500}$th, or about 23 in. to a mile. The uncultivated districts of the Highlands and islands are done on the 6 in. scale—the publication of the island portions, by the more rapid progress of zincography, being now quite completed. In these maps the height of every hill is given, and its contour is drawn, and, as showing the careful nature of this important work, it may be mentioned that all antiquities are noted, and, by the employment of different styles of lettering, their Roman, Druidical, Saxon, or Norman character is indicated. The engraving of the 1 in. general map, which, from the nature of the processes employed, is necessarily much slower, is now proceeding apace, and the sheets are being quickly published.—*Engineering.*

MANUFACTURE OF ARTIFICIAL PARCHMENT.—Messrs. Herold & Gawalowski, of Brunn, make as follows, a strong, artificial parchment, impermeable by water, and capable of serving for the diaphragm in osmotic operations on solutions of impure sugar, &c. The woolen or cotton tissues are freed, by washing, from the foreign substances, such as gum, starch, &c., which may cover them. They are then placed in a bath slightly charged with paper-pulp; and to make this pulp penetrate more deeply, they are passed between two rollers, which slightly compress them. The principal operation consists in steeping the product for a few seconds in a bath of concentrated sulphuric acid, after which it undergoes a series of washings in water and ammoniacal liquor, until it has lost all trace of acid or base. It is then compressed between two steel rollers, dried between two others, covered with felt, and finally calendered, when they are fit for use.

AN interesting experiment on the transmission of power by electricity was made at the Munich Electrical Exhibition recently. Two Gramme dynamos were used, one located in Munich and the other at Meisbach, 35 miles distant. They were connected by an ordinary galvanized iron telegraph wire 4½ millimeters or about one-sixth of an inch in diameter, being, in fact, a telegraph line placed at the disposal of the experimenters by the German Telegraph Administration. A second wire was used instead of the earth for the return circuit. The total resistance of the wire was 950 ohms. The resistance of each dynamo was 470 ohms. The total resistance of the working circuit was therefore, 1,890 ohms. When the generator at Meisbach was driven 2,200 revolutions per minute, 1,500 revolutions per minute were obtained on the receiving dynamo at Munich. The per centage of power utilized at this dis-

tance was therefore $\frac{1}{2500}$ = over 60 per cent. of that expended, and at the time of the experiment a heavy rain was falling, which must have considerably impaired the insulation of the line.

THE CANAL AGE.—Apropos of the movement at present in progress for the construction of the ship canal between Liverpool and Manchester, a writer in an ably-conducted north-county paper very pointedly draws attention to the probability of the remaining years of the nineteenth century being spoken of in history as "The Canal Age," his opinion being that the present indications are in the line of a large extension of inland water carriage by means of canals, and that the problem of quick international communication has now been solved, almost to "finality," by steamships and railways. Whether or not finality has been reached by those two great civilizing agencies, it is undoubtedly the case that the prospects of canalization on a great scale for the immediate future bulk very largely in the eyes both of commercial men and of engineers. Not only is there in hand the project of the Liverpool and Manchester ship canal, with its probable cost of £6,000,000, estimated to make an income enough to pay the shareholders if only a single ship of 4,000 tons pass both ways every day, but there are also various other great inland water-way schemes, of national and international importance, either in hand or actually carried into execution. The sum of £40,000,000 has recently been voted by the French Parliament for inland canalization works, and it is thought that at least five times that sum will have been spent upon such works before the system of inland water carriage in France has been completed. Many of our readers are familiar with the great engineering works which have resulted in the completion of a ship canal connecting the city of Amsterdam with the sea; and they scarcely require to be informed that it has proved to be a remarkable success, commercially and otherwise. Additional canals are likewise in course of construction or projected in Belgium, a country well adapted by nature for such works. Then, going into Prussia, we find that there is a prospect of a speedy beginning with the canal scheme which aims at connecting the Rhine, the Weser, and the Elbe, at an estimated cost of upwards of £7,000,000. Proceeding further east we should notice another proposal which bids fair to become an accomplished fact in the early future, which is a scheme to cut a ship canal to connect the river Danube with the Oder, and thereby joining the Black Sea with the Baltic. But in Russia it is proposed to enter upon even a much larger canal scheme, to wit, one to connect the river Dneister with the Vistula, and thereby to bring the great ports of Odessa and Dantzic into direct communication. One of the probabilities of the next few years is an Egyptian project, namely, a great inland water-way to rival the Suez Canal; and a ship canal through the Isthmus of Panama may be regarded as one of the certainties of the immediate future. More or less similar schemes are likewise contemplated in other parts of the world—in Canada, in Southern Europe, Southern Asia, etc.—*Engineering.*

IN a report to the Board of Trade on a collision which occurred on the 30th ult. at Crewe Station on the London and Northwestern Railway, Major Marinden says, "In conclusion it should be observed that, as is usually the case when a driver has more than one class of brake at his command, some valuable time seems to have been lost in the application of them. Had the train been fitted with a quickly-acting continuous brake, fitted to the engine and all the vehicles, the whole train might have been braked by the same simple action which was taken by the driver when he applied his engine steam-brake, and the speed would have been very much less than it was, even if the train had not been altogether stopped before any collision took place," and in concluding his report on the collision which occurred on the 28th of September at Chester, on the London and Northwestern Railway, when fourteen passengers and Post-office sorters and the front guard of the mail train were injured, says:—"It is worthy of remark that, although there were sufficient time for the driver to reverse his engine, and to whistle for the guard's brake, for his fireman to apply the tender-brake, and for the front guard to skid the wheels of his brake-van by means of his hand-brake, yet no attempt was made by either driver or guard to apply the patent chain brake, which, according to the regulations of the company, is to be used as an emergency brake only. This is quite in accord with my experience of the action of the servants of this company in other like instances, and it seems to me that it furnishes a strong argument in favor of the habitual, and not the casual, use of whatever continuous brake is adopted by any railway company."

ONE of the chief, or the greatest difficulty met with in plating iron an d steelwith other metals has been that, in the absence of a flux, the oxidation of the metals during the preliminary process of heating prevented a perfect weld. But it is stated that by the use of a metallic chloride vaporizable at a red heat, such as chloride of zinc or muriate of ammonia, a perfect union may be obtained, and this is better assured if a coat of tin-foil be placed between them as a welding medium. A flat bar of iron or steel treated in this manner and enveloped in the plating metal is brought to a red heat and drawn to any required dimension between cold rolls, and is said to be in use in Prussia.

AT a recent meeting of the Berlin Polytechnic Society, Dr. Franke exhibited specimens of a condensed fuel prepared with a solution of dextrine and saltpetre, which, according to the quantity of the latter substance, burns from six to twelve hours, and is considered to be specially applicable to the heating of railway carriages and to other similar purposes. Such a fuel will, doubtless, save some trouble in German railway carriages, which are heated by cakes of a low-burning fuel placed in small horizontal boxes in each carriage, the carriages having iron frames and panels. The heating is much more efficient and less inconvenient to travelers than our makeshift hot-water boxes.